Topics 1

- The main road we'll follow to create experimental designs for real-world problems is to use **model-driven computer-generated designs**
- Side roads and detours for special cases will include
  - Constraints – avoiding unworkable conditions
  - Augmenting designs – adding on to existing trials
  - Repairing broken designs - augmenting with constraints
  - Hard-to-change factors – split plot designs and analysis
  - Different optimality criteria – when one might not use the default
  - Bayesian Optimal design
  - Covariate designs – using historical data as a candidate set
  - Mixture/formulation designs

    1) Model-driven, 2) Space-filling, 3) Covariate candidates, 4) Functional Data response

jmp

Topics 2

- Modern screening designs
  - Definitive screening (DSD)
  - DSD comparison to classic Plackett-Burman (PB) and Fractional Factorial (FF)
  - Orthogonal and nearly orthogonal arrays (OA & NOA)
  - Orthogonal mixed-level (OML)
  - Group orthogonal supersaturated (GOSSD)
- Space-filling designs for computer simulation experiments
  - Latin HyperCube (LHC)
  - Fast Flexible Filling (FFF)
  - BEAST mode (<u>B</u>oundary <u>E</u>xploration and <u>A</u>daptive <u>S</u>ampling <u>T</u>echnique)

Topics 3

- Data Transformations to help improve analyses
  - More often on *responses* to normalize the data and to prevent nonsensical predictions
  - Sometimes on the *factors* to add scientific understanding to analysis
- Checkpoints – where?
- Non-linear designs
  - Sequential binary response testing
- Accelerated Life Test (ALT) design
- Measurement System Analysis (MSA) design
- Analysis of Functional Data (curve or spectra) as a response
- Covering Arrays – efficient fault detection

Why use DOE?

Quicker answers,
Lower costs,
Solve bigger problems

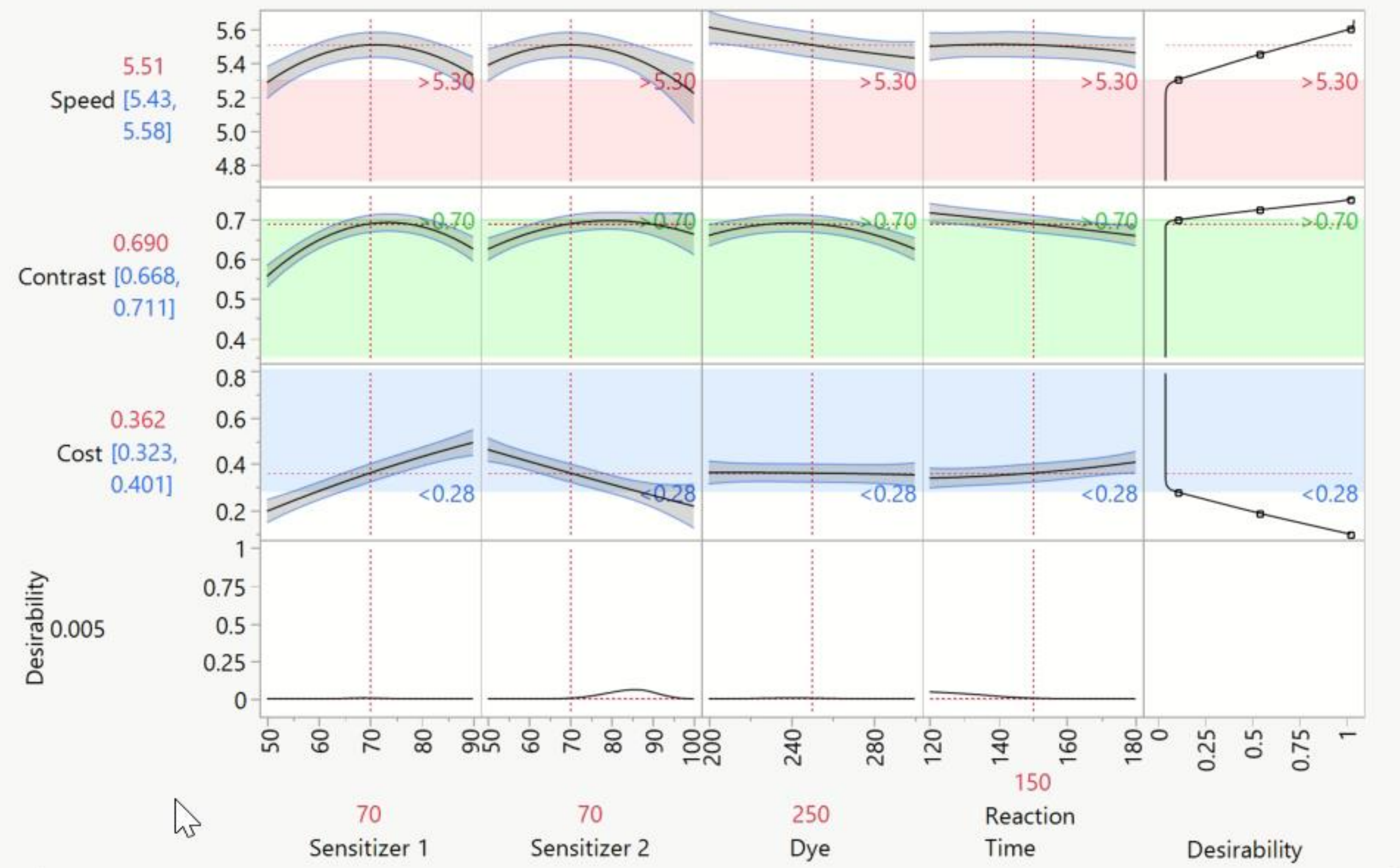
- More rapidly answer "what if?" questions
- Identify important factors when faced with many
- Do sensitivity and trade-space analysis
- Optimize across multiple responses
- Characterize the operating region
- By running efficient subsets of all possible combinations, one can – for the same resources and constraints – *solve bigger problems*
- By running sequences of designs one can be as *cost effective as possible* and *run no more trials than needed* to get a useful answer

Agent Fate 10,000+, USAF Sim Study 648

DOE with
4 Factors,
3 Responses,
& RSM Model

7/4 / 27/0	Sensitizer 1	Sensitizer 2	Dye	Reaction Time	Speed	Contrast	Cost
	90	90	300	180	5.59	0.73	0.73
	50	50	200	120	4.79	0.36	0.17
1	50	50	250	120	5.36	0.616	0.198
2	50	50	200	180	5.39	0.537	0.175
3	90	70	200	120	5.31	0.623	0.447
4	50	90	200	150	5.13	0.431	0.177
5	70	70	250	180	5.37	0.643	0.445
6	50	90	300	120	4.79	0.375	0.231
7	90	90	200	180	5.45	0.626	0.471
8	90	50	250	150	5.00	0.470	0.670
9	50	50	300	150	5.22	0.478	0.283
10	70	90	200	120	5.41	0.668	0.226
11	90	90	250	120	5.33	0.734	0.310
12	50	50	250	120	5.32	0.574	0.257
13	70	50	200	150	5.49	0.596	0.456
14	50	70	250	180	5.22	0.558	0.166
15	70	70	250	150	5.57	0.689	0.390
16	90	90	300	150	5.26	0.653	0.226
17	70	70	250	150	5.47	0.688	0.356
18	70	70	300	120	5.42	0.657	0.337
19	50	70	200	120	5.43	0.518	0.222
20	50	50	300	150	5.15	0.505	0.287
21	90	70	200	120	5.33	0.661	0.457
22	50	90	300	120	4.97	0.411	0.191
23	90	50	300	120	5.09	0.492	0.588
24	90	50	300	180	5.03	0.358	0.733
25	70	70	250	150	5.59	0.707	0.318
26	70	90	300	180	5.25	0.605	0.290
27	50	90	200	150	5.24	0.476	0.177

Prediction Profiler

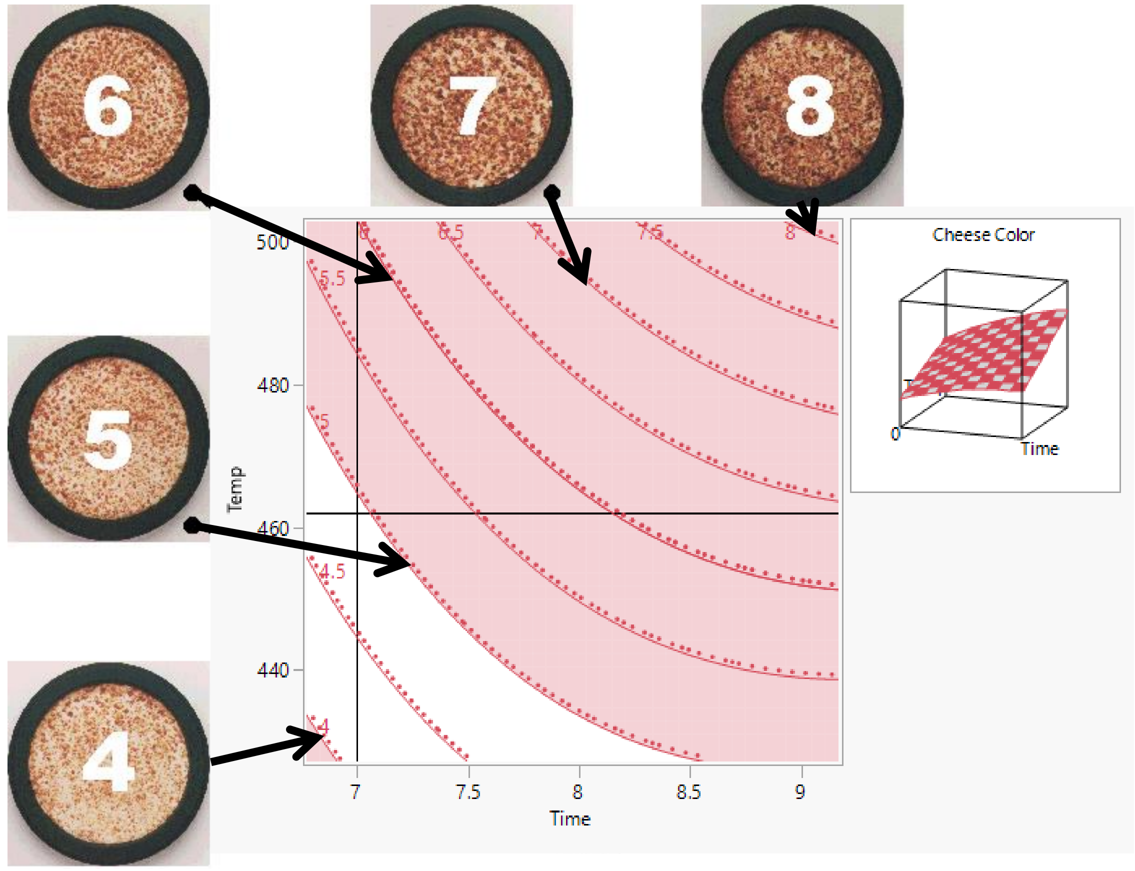


Remembered Settings

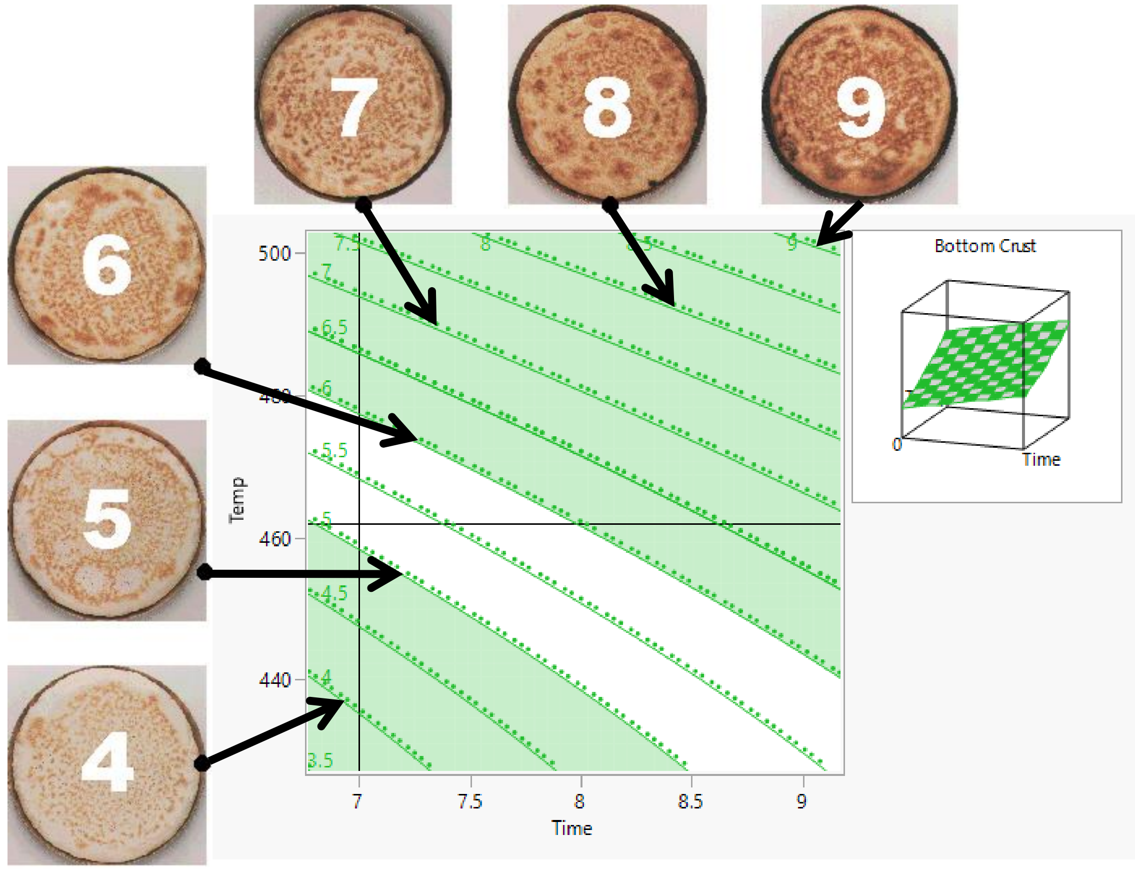
Setting	Sensitizer 1	Sensitizer 2	Dye	Reaction Time	Speed	Contrast	Cost	Desirability
Equal Importance Opt	80.8	91.3	251	120	5.35	0.747	0.250	0.348
Mid Point Settings	70.0	70.0	250	150	5.51	0.690	0.362	0.005
Cost 6X Speed & Contrast	84.0	93.7	283	120	5.29	0.725	0.199	0.214
Opt Spd3X-Cntr1X-Cost6X	82.0	90.7	287	120	5.33	0.718	0.221	0.264

Multiple Response Optimization – Best Trade-Off of Three Target Values

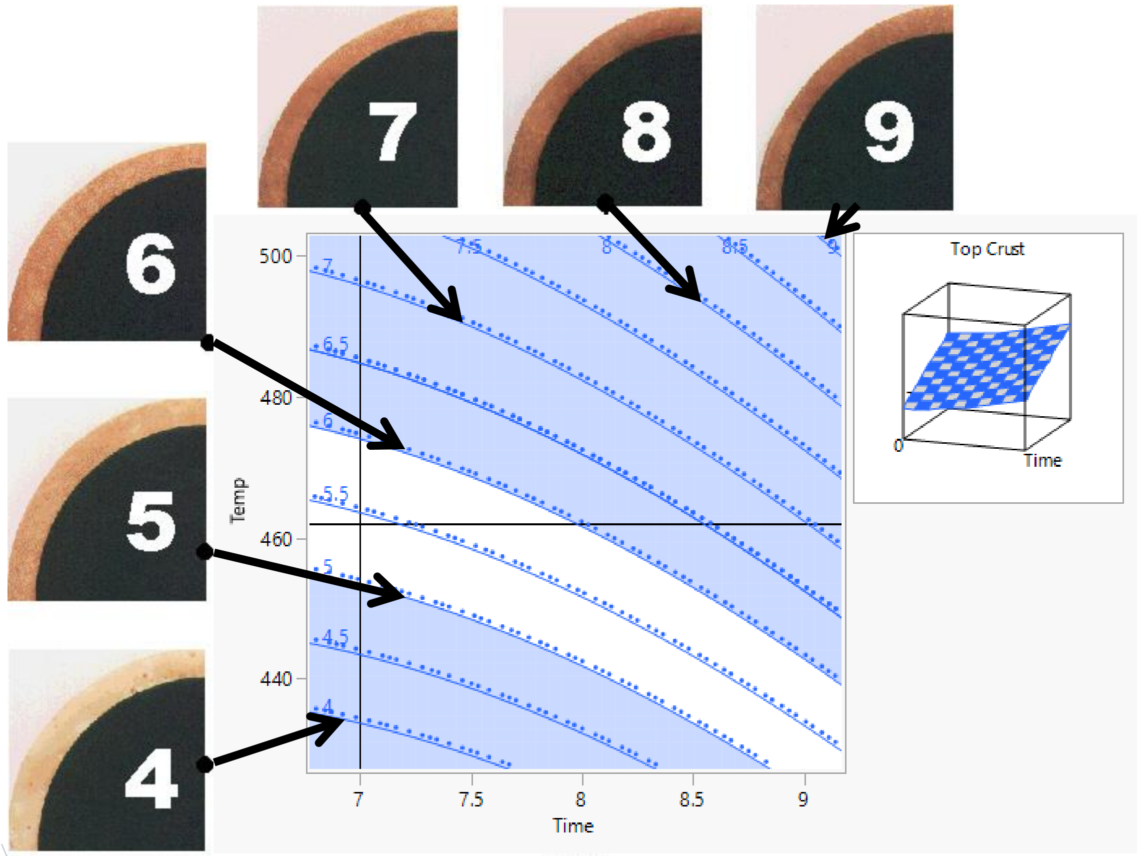
Cheese Color
Target Value
4.5 ± 0.5

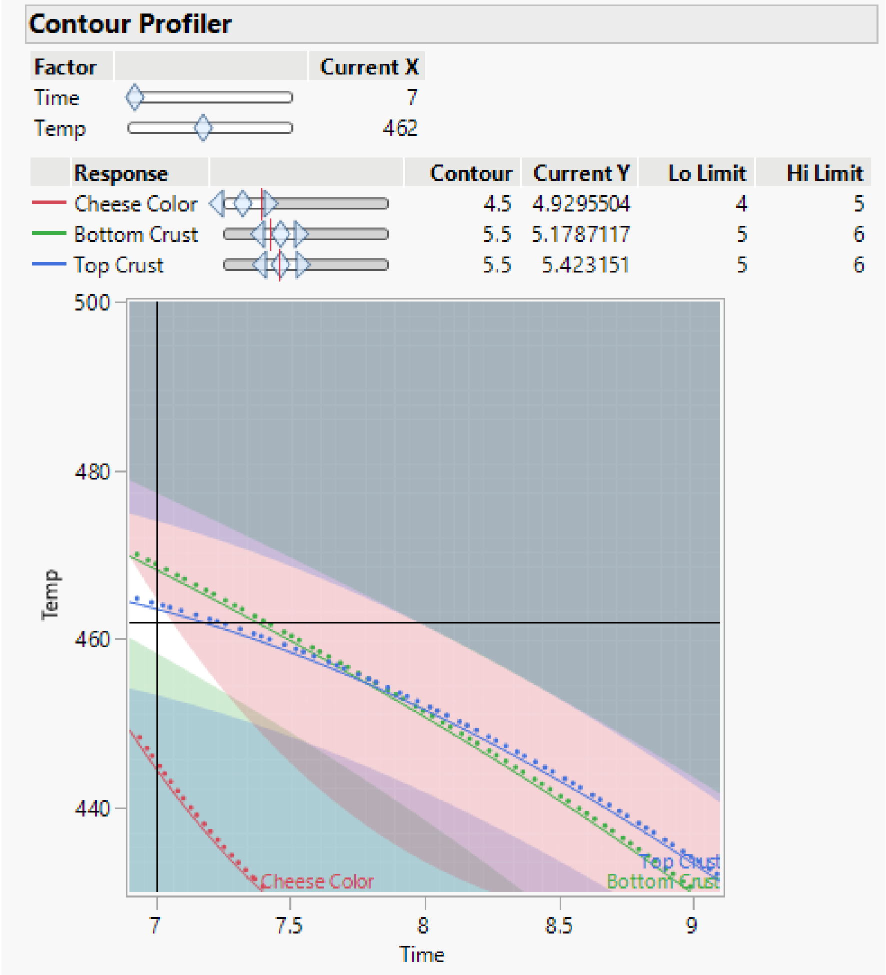


Bottom Crust
Target Value
5.5 ± 0.5



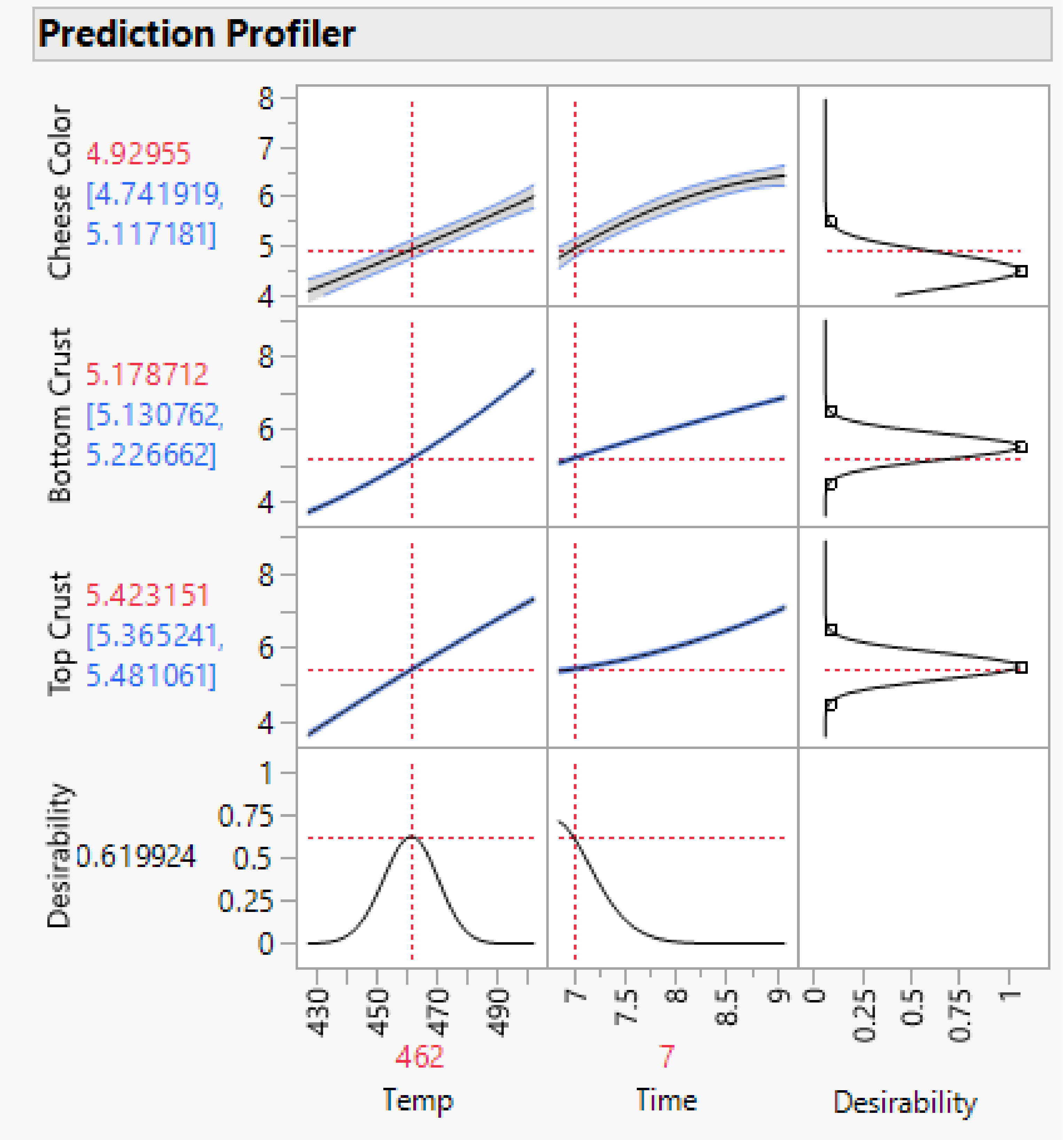
Top Crust
Target Value
5.5 ± 0.5

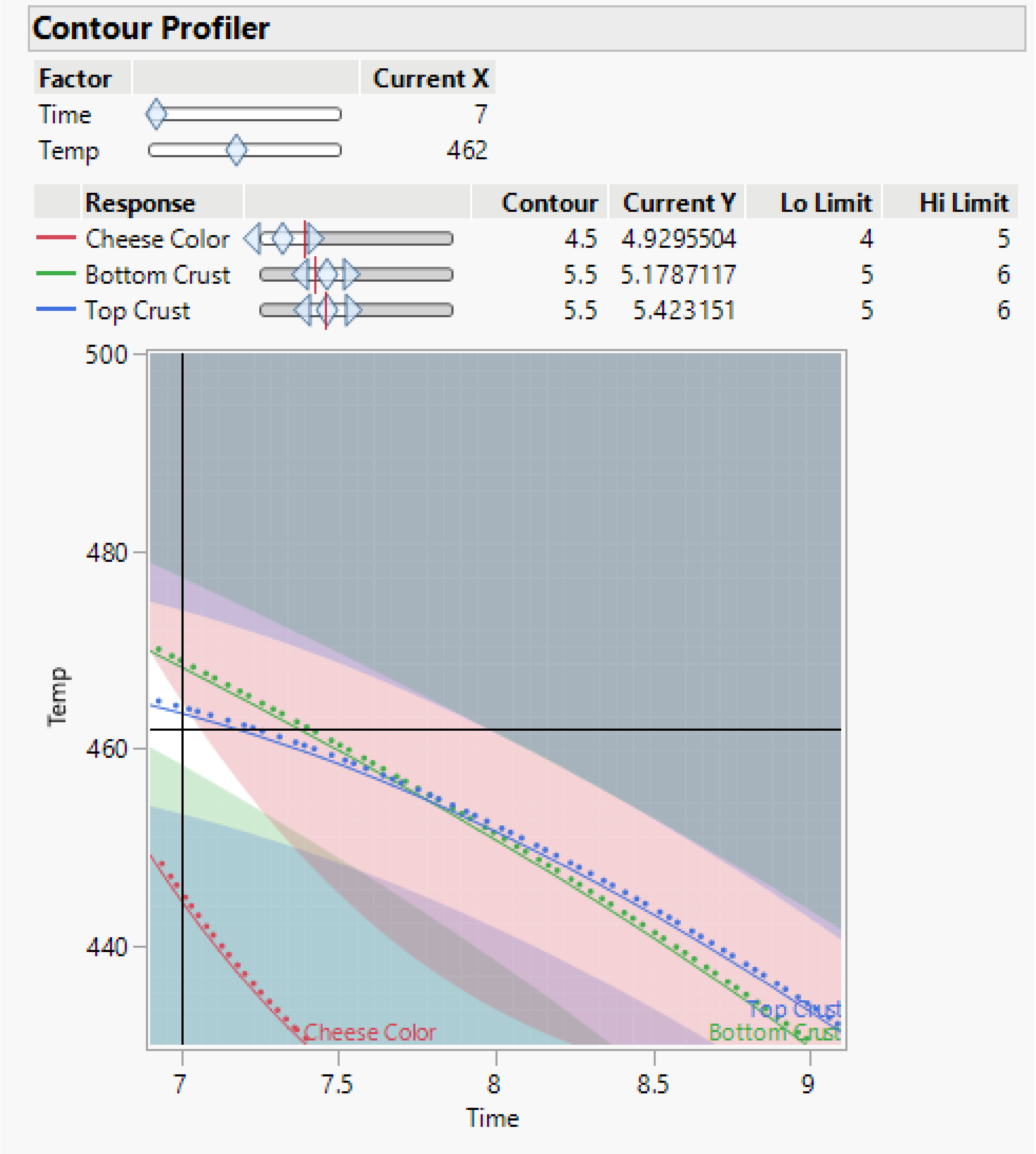


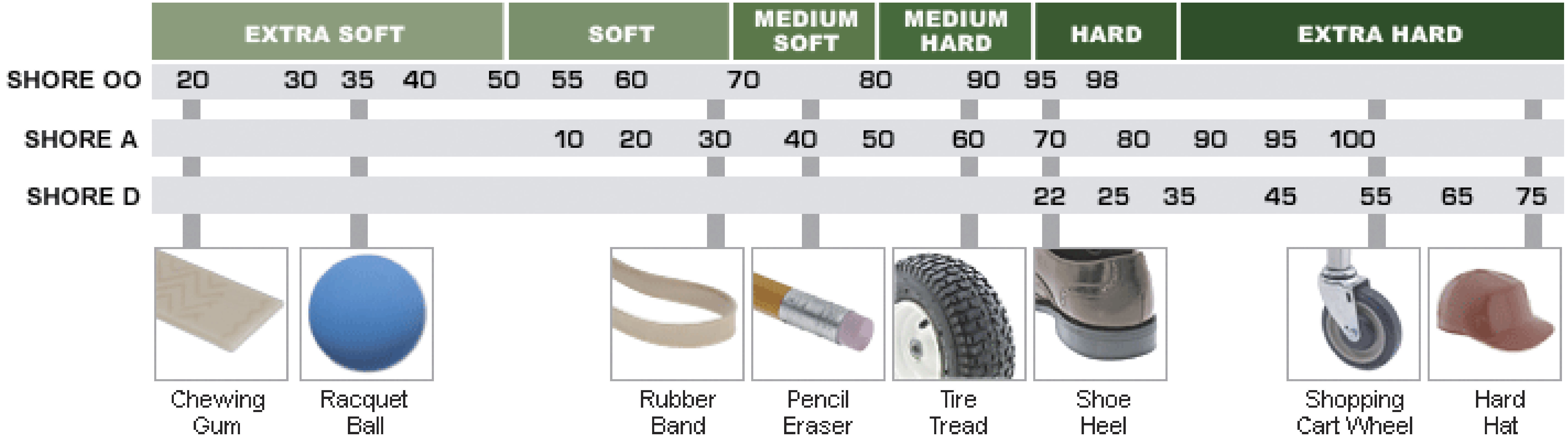


Overlay of Contour Plots
Acceptable Region is White

Multiple Response Optimization – Best Trade-Off of Three Target Values



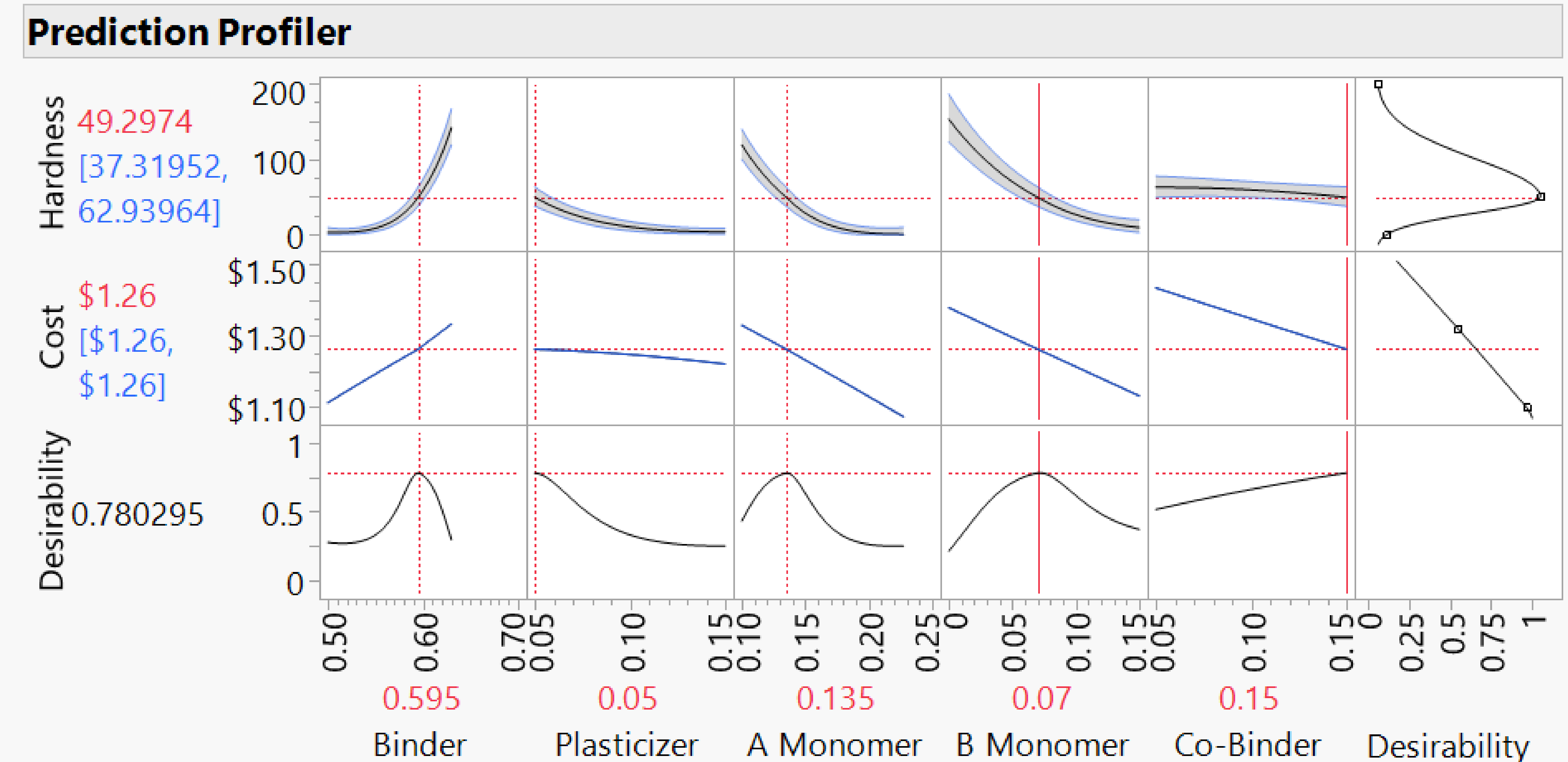




Mixture DOE:
5 Formulation Components & 2 Responses

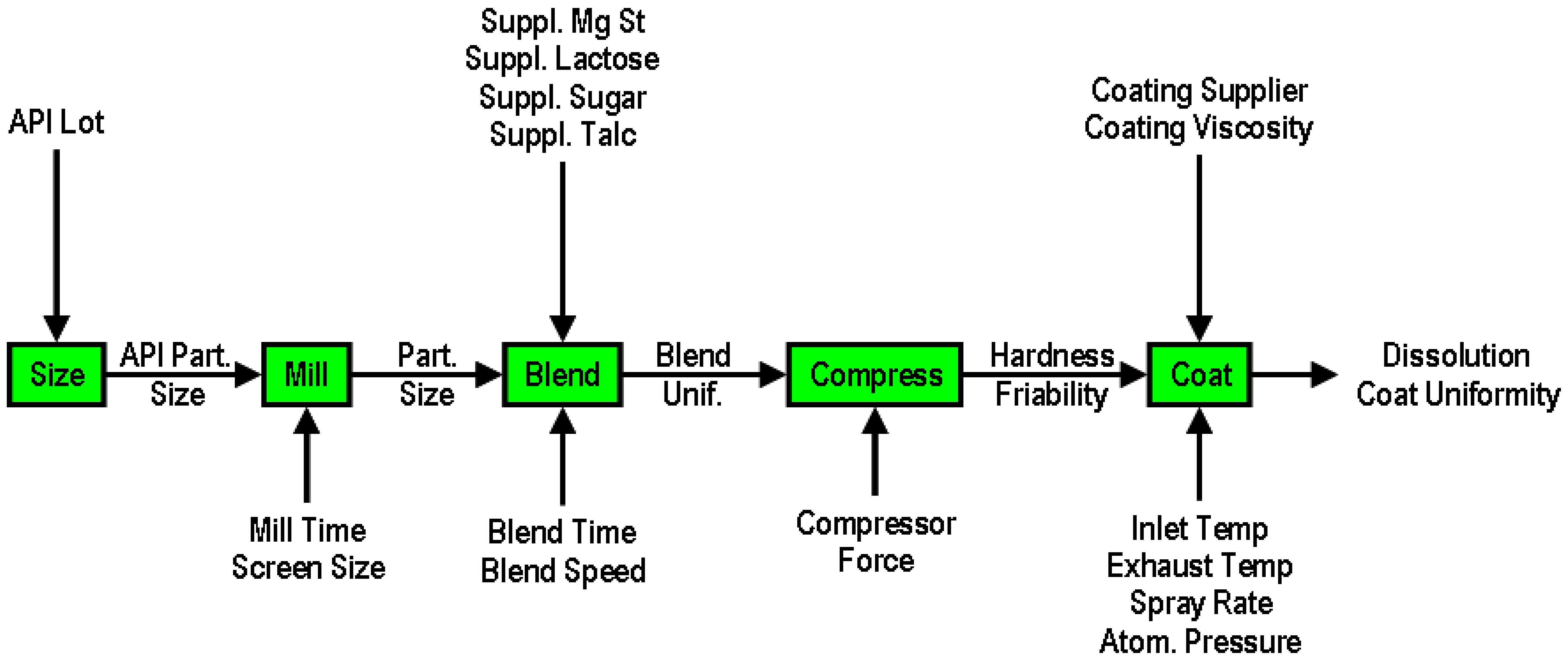
DOE results allow customers to choose *Hardness* they want, and then you manufacture product at the lowest *Cost*.

Set desirability to **target** *Hardness* at 50 Shore units, and to **minimize** *Cost*.



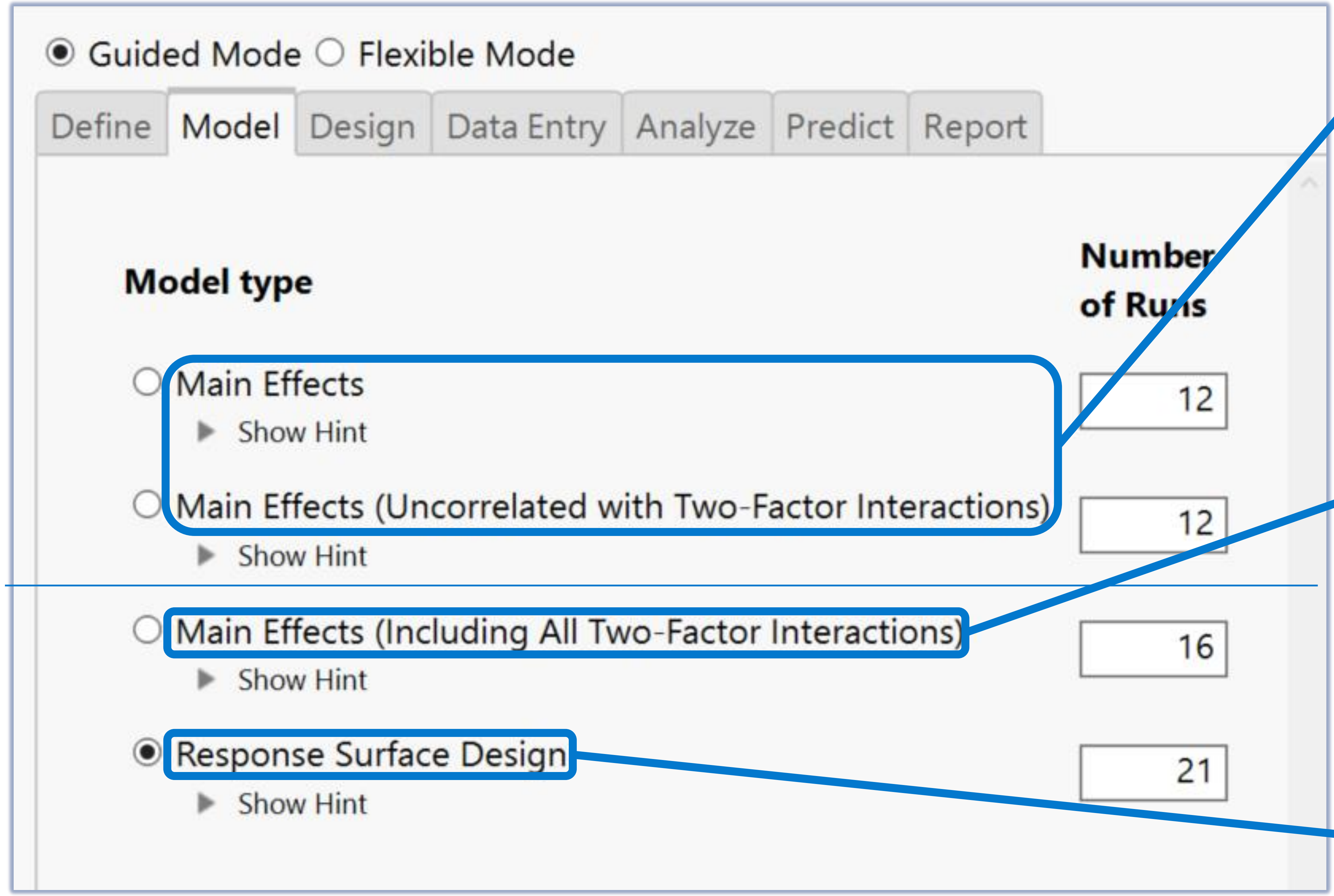
Classic Definition of DOE

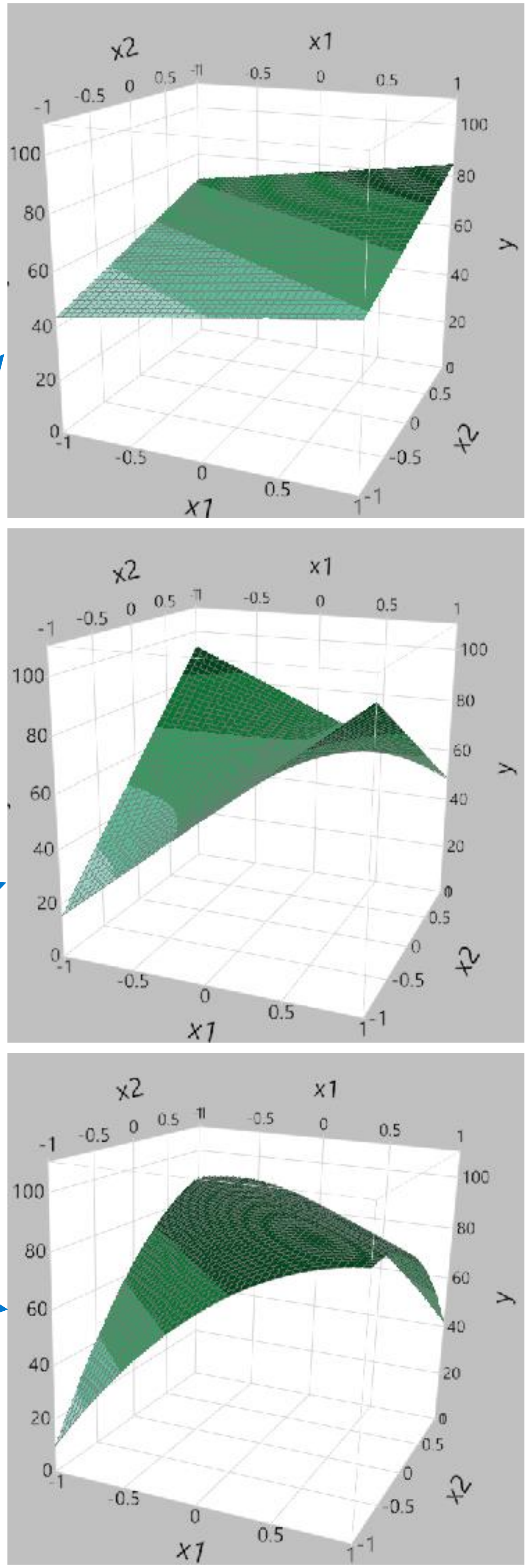
Purposeful control of the inputs (factors) in such a way as to deduce their relationships (if any) with the output (responses).

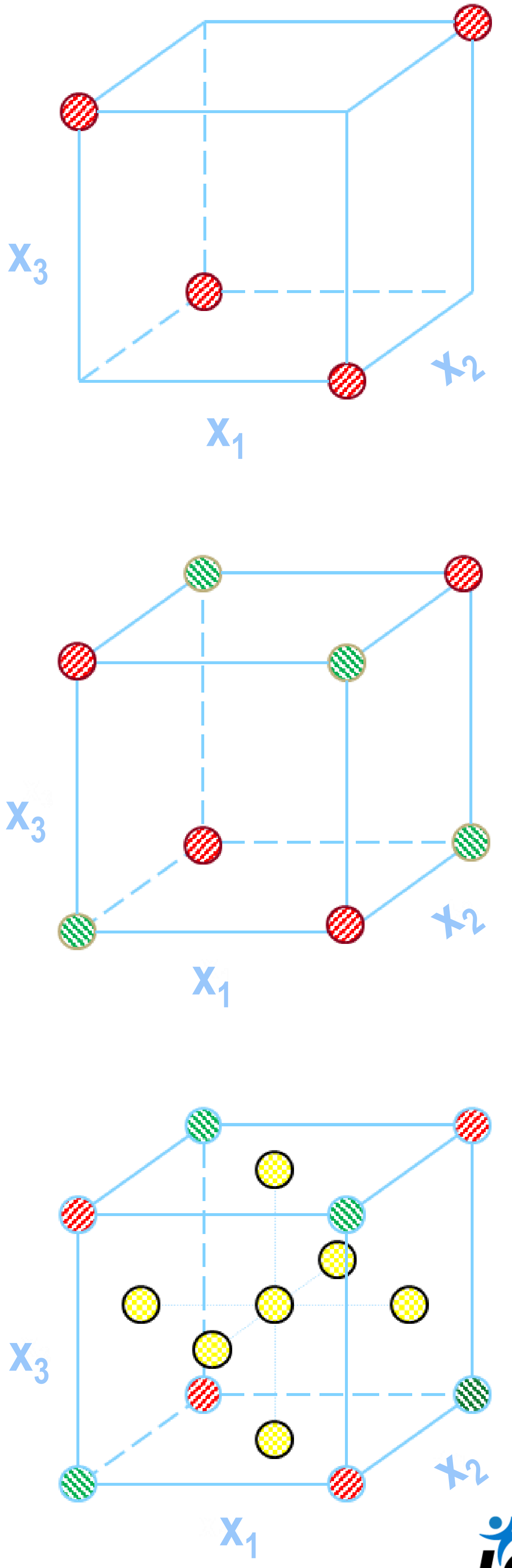


Alternative Definition of DOE

A DOE is a specific collection of trials run to support a *proposed* model.







As complexity to be supported increases, so do the number of runs

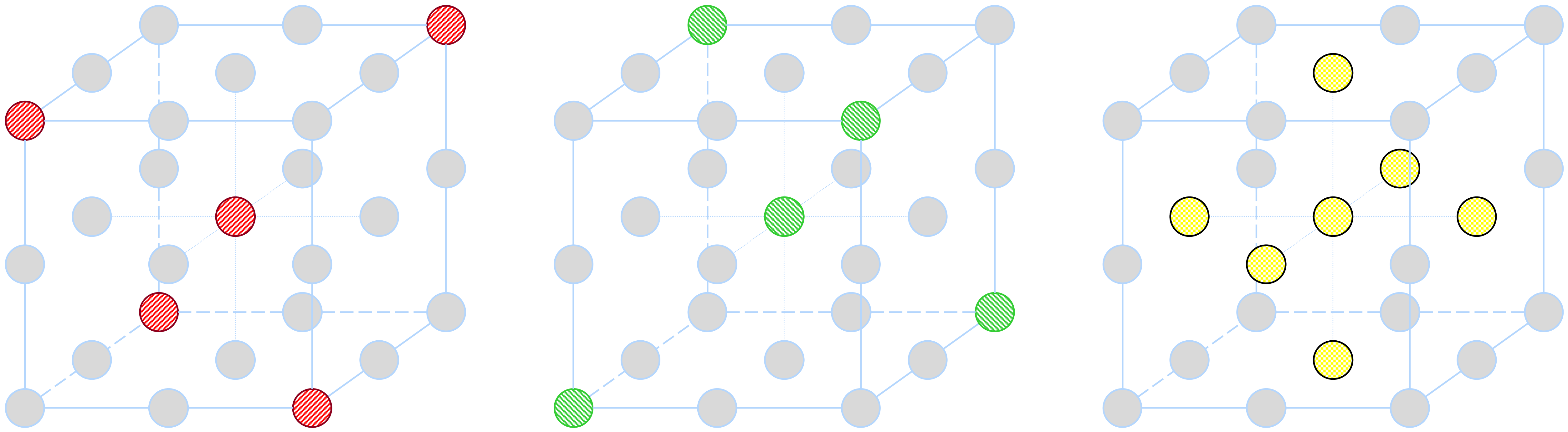
Alternative Definition of DOE

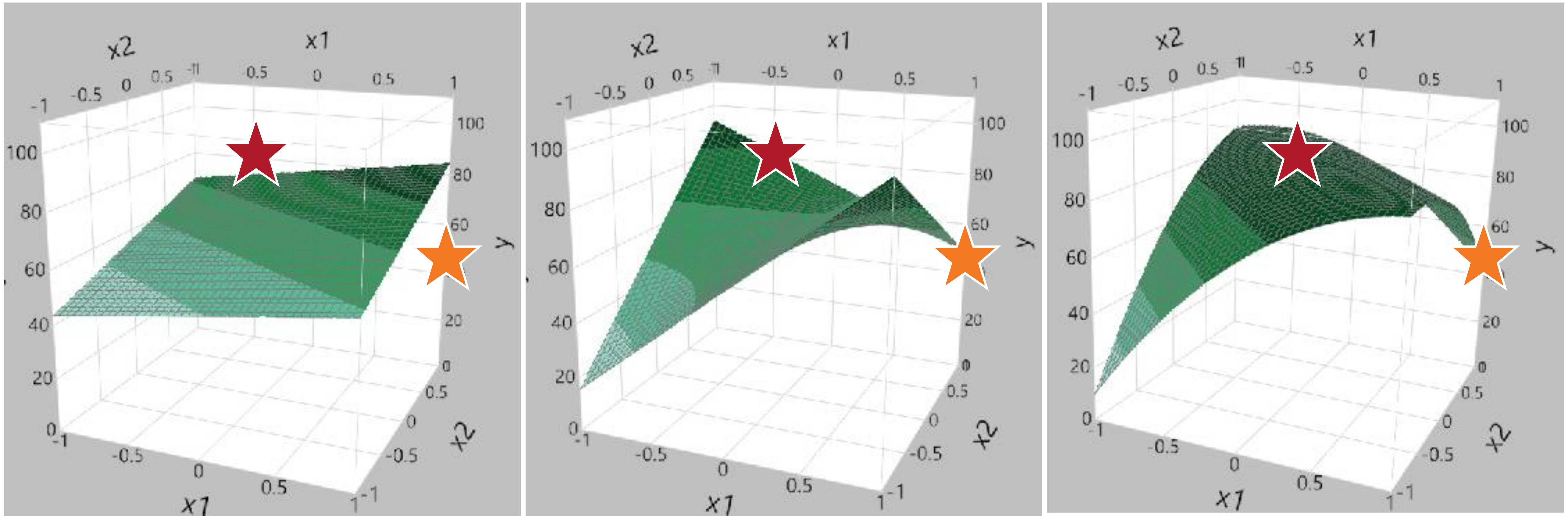
A DOE is a specific collection of trials run to support a *proposed* model.

- If proposed model is ***simple* -** main effects or *1st order* terms (x_1 , x_2 , x_3, etc.) - the design is called a *screening* DOE
  - Goals include ranking factor importance, or "finding a winner" quickly, but NOT making predictions
  - Used with many (> 6?) factors at start of process characterization
- If the proposed model is *more complex*, with *2nd order* terms so that it includes two-way interaction terms (x_1x_2, x_1x_3, x_2x_3, etc.) and in the case of continuous factors, squared terms (x_1^2, x_2^2, x_3^2 , etc.), the design is called a *response-surface* DOE
  - Goals include developing a predictive model of the process, or characterizing the operating region of the design space
  - Used with a few (< 6?) factors often after a screening DOE

Classic Response-Surface DOE in a Nutshell

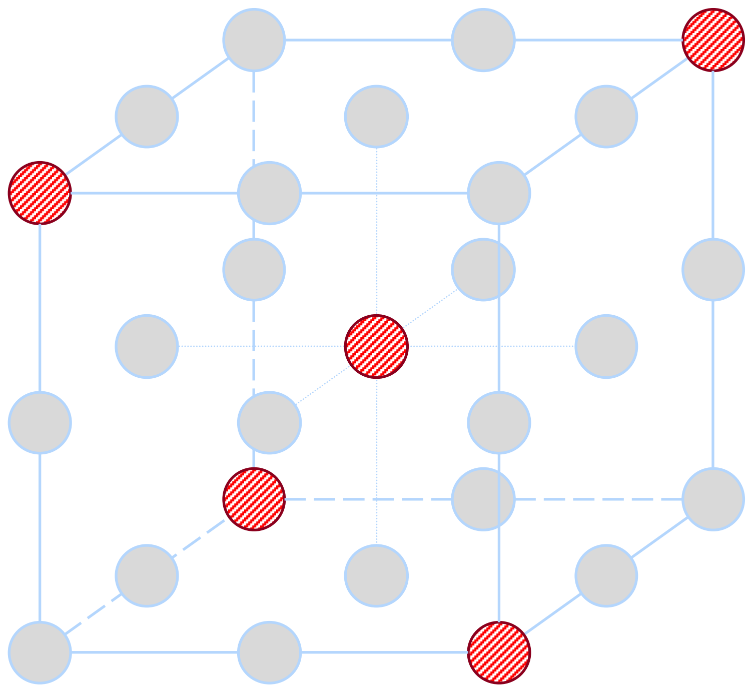
Block 1 | Block 2 | Block 3





Polynomial Models Used to Calculate Surfaces

Block 1

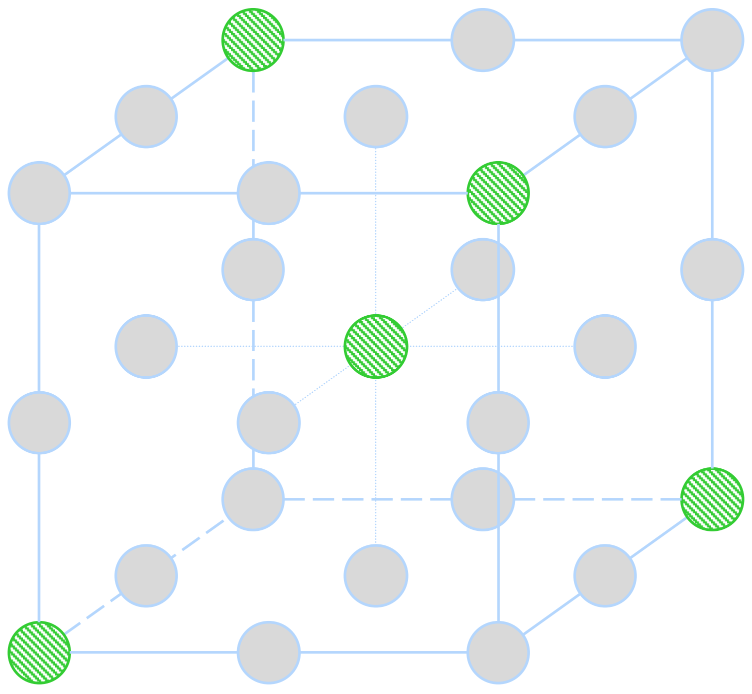


$$y = a_0 + a_1x_1 + a_2x_2 + a_3x_3$$

Run this block 1st to:

(i) estimate the main effects
(ii) use center point to check for curvature.

Block 2



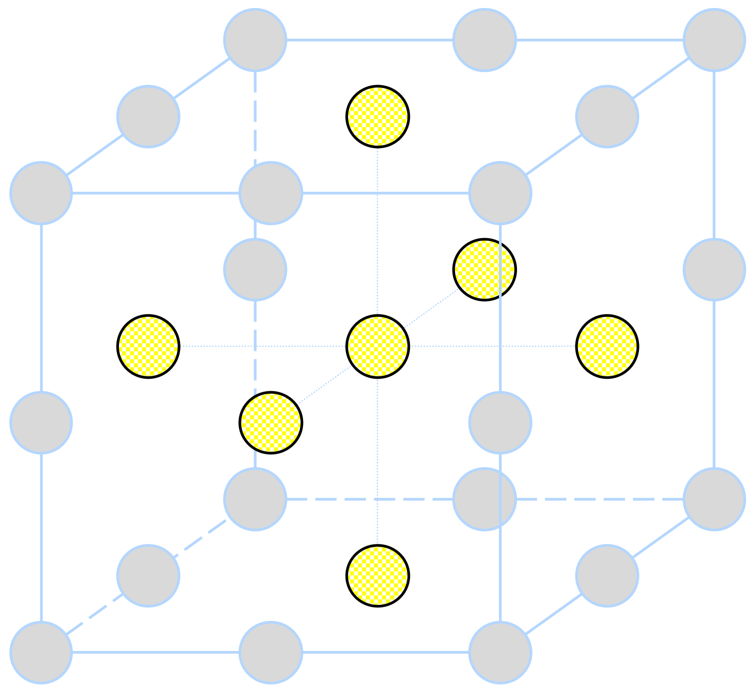
$$y = a_0 + a_1x_1 + a_2x_2 + a_3x_3$$

$$+ a_{12}x_1x_2 + a_{13}x_1x_3 + a_{23}x_2x_3$$

Run this block 2nd to:

(i) repeat main effects estimate
(ii) check if process has shifted
(iii) add interaction effects to model <u>if needed.</u>

Block 3



$$y = a_0 + a_1x_1 + a_2x_2 + a_3x_3$$

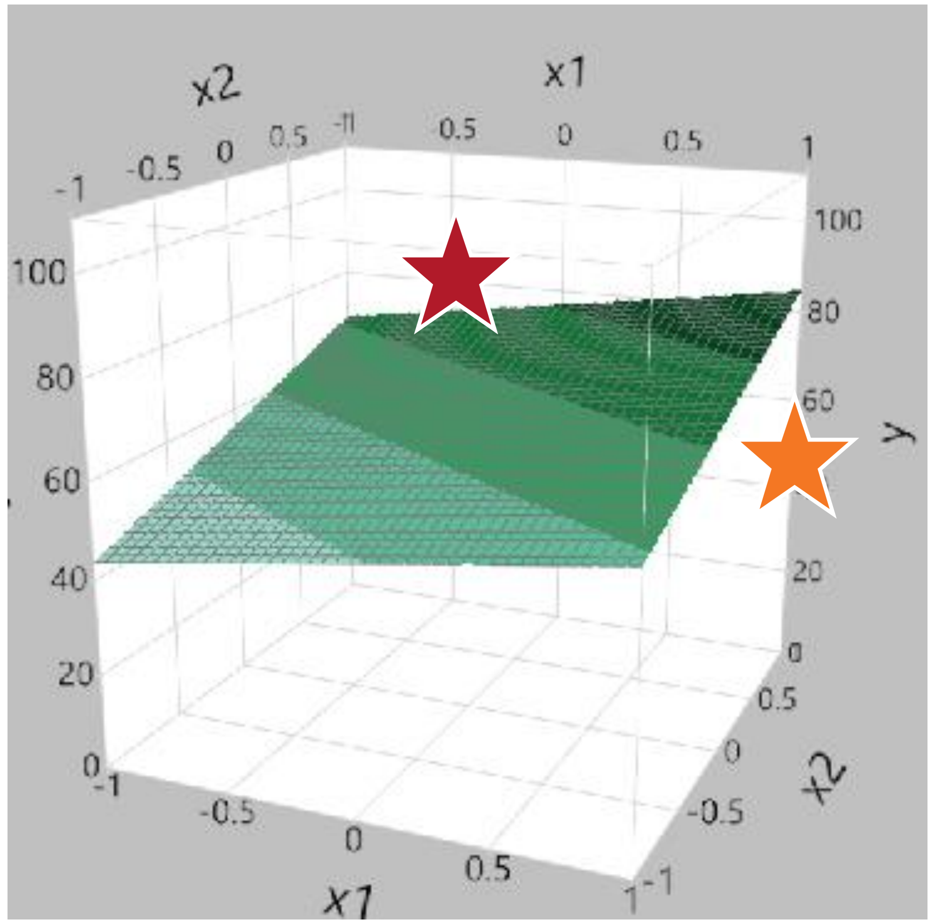
$$+ a_{12}x_1x_2 + a_{13}x_1x_3 + a_{23}x_2x_3$$

$$+ a_{11}x_1^2 + a_{22}x_2^2 + a_{33}x_3^2$$

Run this block 3rd to:

(i) repeat main and interaction effects estimate
(ii) check if process has shifted
(iii) add curvature effects to model <u>if needed.</u>

Quadratic model is not much bigger than *Interaction* model.
If you have continuous factors, choose full 2nd order, *Quadratic*.

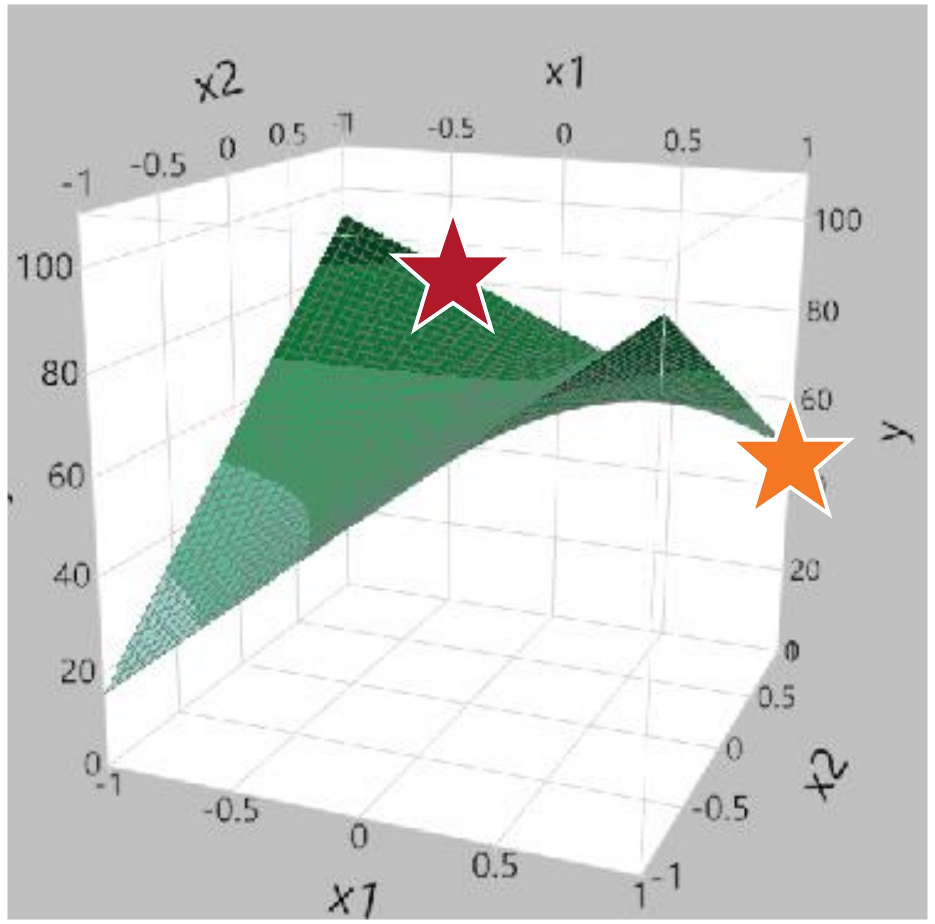


1st Order

$y = a_0 + a_1x_1 + a_2x_2$

For k factors there are
k main effects

3-factor Linear Model has 4 terms (8 corners)
6-factor Linear Model has 7 terms (64 corners)
10-factor Linear Model has 11 terms (1K corners)
20-factor Linear Model has 21 terms (1M corners)



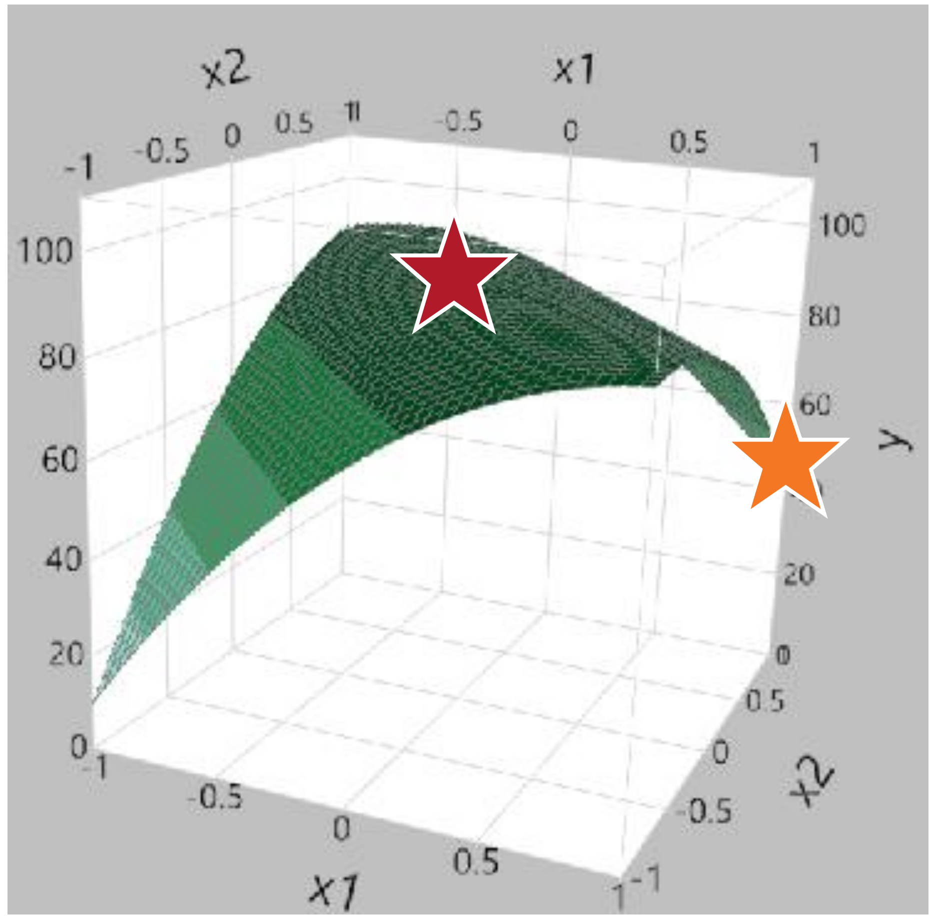
2nd Order

$y = a_0 + a_1x_1 + a_2x_2$

$+ a_{12}x_1x_2$

For k factors there are
k(k-1)/2 interaction effects

3-f Interaction Model has 7 terms (2X ME)
6-f Interaction Model has 22 terms (3X ME)
10-f Interaction Model has 56 terms (5X ME)
20-f Interaction Model has 211 terms (10X ME)



Full 2nd Order

$y = a_0 + a_1x_1 + a_2x_2$

$+ a_{12}x_1x_2$

$+ a_{11}x_1^2 + a_{22}x_2^2$

For k factors there are
k squared effects

	3^k combinations
3-f Quadratic Model has 10 terms (2.5X ME)	27
6-f Quadratic Model has 28 terms (4X ME)	729
10-f Quadratic Model has 66 terms (6X ME)	59K
20-f Quadratic Model has 231 terms (11X ME)	3.5B

If no squared terms, then optimum can ONLY be a corner!

6-Step DOE Process

Describe → Specify → Design → Collect → Fit → Predict

Describe: Identify goal, responses, and factors.
Ranges require SME!

Specify: Identify effects for an assumed model.
Propose 1^{st} or 2^{nd} order?

Design: Generate a design and evaluate it for suitability.

Collect: Run trials using design settings. Measure response for each run.

Fit: Determine a model that best fits experimental data.

Predict: Use the model to optimize factor settings or to predict process performance.

Loads of Questions to Ask

Describe
1. What is the goal of the experimentation?
2. How do you measure success?
3. What response variables do you measure?
4. If more than one response needs to be characterized for your process, what are the relative levels of importance?
5. What kind of control factors do you have?
   1. Continuous (Quantitative) – varies over a range.
   2. Categorical (Qualitative) – varies as different levels.
   3. Discrete Numeric – analyzes like a continuous factor, but only available at discrete levels, like an ordinal categorical factor.
   4. Mixture or formulation factor – behaves like a continuous factor, but all mixture component proportions are constrained to sum to (typically) 1.00.
   5. Blocking factors – groupings of trials such as *day* or *batch* that should not have an effect. We add blocking factors to see if the process shifts between groups as an indicator of unknown or lurking factors being correlated with the blocks.
6. Over what ranges does it make sense to operate the control factors?
   1. Too bold may break the process.
   2. Too timid may not generate a sufficiently large effect.
   3. Don't know? Involve subject matter experts on the process.
7. Are there potentially important factors that can't be controlled?
8. Can any uncontrollable factors be monitored so that the settings can be captured and recorded (e.g., ambient temperature, humidity, operator at time of trial)? These can be treated as covariate factors.
9. Does the process drift over the course of the day or period being measured?
10. How many trials can be run in a day? Will multiple days be required?
11. Do you typically run control samples for this process?
12. Will trials be run in batches or groups?
13. Are there any hard-to-change factors, and if so, which ones?
14. How many devices do you have of each type?
15. Do you have historical data that can be "data mined" for possible factors and to better understand factor ranges?
16. Are these *real* experiments or are they *computer simulations*?
17. If simulations, are they deterministic (same answer every time), or stochastic (randomness built in so answer is slightly different each time)?

Specify
1. Are you looking to identify important main effects from a large set (e.g., 6+) of factors?
2. Are you looking to build a predictive model with which to characterize and optimize a process?

   NOTE: These two questions determine if the proposed model will be 1^{st} order or 2^{nd} order.

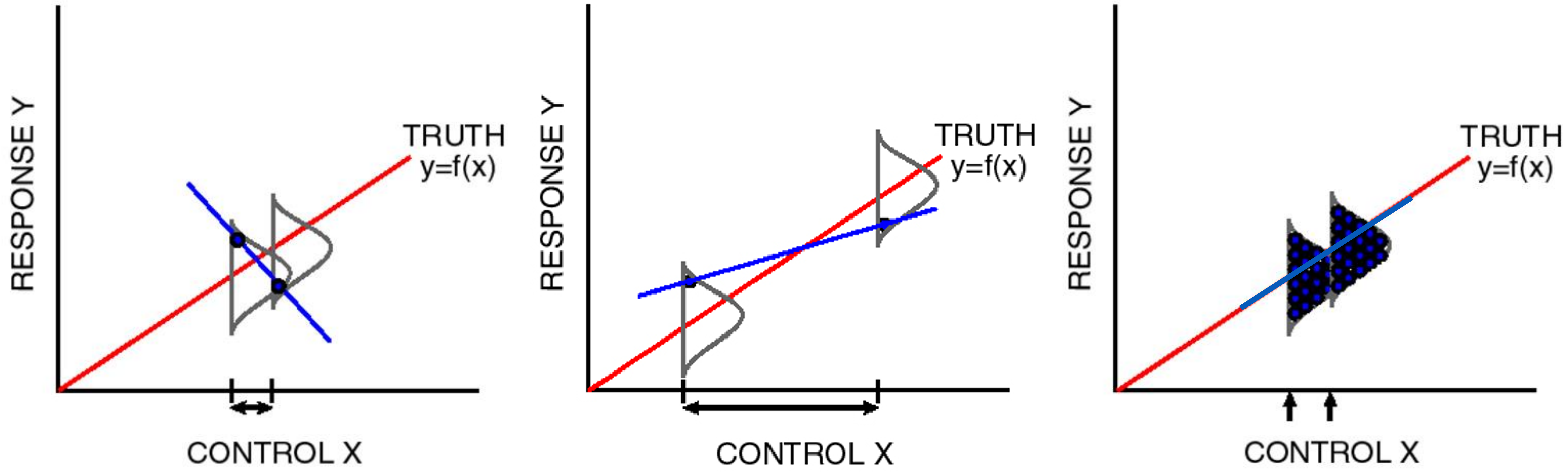
Design
1. What is your budget?
2. What is your deadline?
3. Does every trial cost about the same (i.e., take the same amount of time to setup and run)?
4. Does getting setup to run the first trial cost substantially more than running the next few? Whenever initial setup is large, consider adding extra trials (replicates or especially checkpoints) while they are cheap to run.
5. Do any combinations of variable settings cause problems (e.g., unsafe, too costly, breaks the equipment/process, impossible to achieve)?
6. Will you need to constrain the design space or disallow certain combinations of factor settings?
7. If you run the same process on separate days, do you ever get obviously/surprisingly different results?
8. Do you have past records of replicated trials for each response?
   1. Are the replicate trial response values close together or spread out over time?
   2. How big is the variability for each response? That is, what is the standard deviation or root-mean squared error (RMSE) of the response?
9. How tiny of a difference for each response is considered practically important?
10. For each response, do you think you are looking for tiny differences in big variability (hard to do because lots of replication is needed) or big differences in small variability (easy to do)?
11. What is the desired level of confidence in detecting effects? This is typically 95%, which leads to setting alpha at 0.05 (Type I error).
12. What are acceptable levels of power for the various types of effects (main, interaction, quadratic, categorical levels)? **NOTE:** This is the desired level of confidence in NOT missing an effect if it is real. It is typically 0.8 for main effects and interactions, and less for quadratic effects (Type II error).
13. How hard is it to come back later to run checkpoint trials? Can you build in checkpoint trials now – especially if they are inexpensive to run? If so, where?
    1. Your guess at where the best performance will occur.
    2. Your guess at where the poorest performance will occur.
    3. Your boss' opinion as to where to run the process.
    4. Add a trial to support the next higher model.
    5. Some points outside the design region.
14. What trial do you think is most likely to break the design? **NOTE:** Perhaps run that trial first.

Readable List in Blog

[Turn many small decisions into one big process optimization success (jmp.com)]

Timid vs. Bold Range Settings

Boldness overcomes the need for large sample size



Real-World Design Issues

Reasons why classical designs likely will not work…

Making Designs Fit the Problem –
NOT Making Problems Fit the Designs!

- Work with these different kinds of control variables/factors:
  - Continuous/quantitative? (Finely adjustable like *temperature*, *speed*, *force*)
  - Categorical/qualitative? (Comes in types, like material = *rubber*, *polycarbonate*, *steel* with mixed # of levels; 3 chemical agents, 4 decontaminants, 8 coupon materials…)
  - Mixture/formulation? (Blend different amounts of *ingredients* and the process performance is dependent on the *proportions* more than on the amounts)
  - Blocking? (e.g. "lots" of the same raw materials, multiple "same" machines, samples get processed in "groups" – like "eight in a tray," run tests over multiple days – i.e. variables for which there *shouldn't* be a causal effect
- Work with combinations of these four kinds of variables?
- Certain combinations cannot be run? (too costly, unsafe, breaks the process)
- Certain factors are hard-to-change (temperature takes a day to stabilize)
- Would like to add onto existing trials? (really expensive/time consuming to run, or by adding constraints can repair broken design)

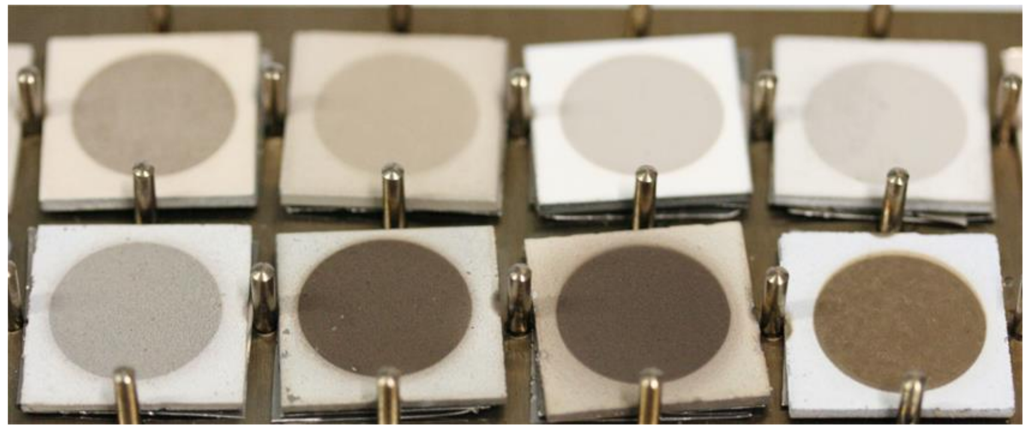
Categorical Factors and Responses

Factors

- Agent
  - Agent 1
  - Agent 2
  - Agent 3
- Decontaminant
  - Decon 1
  - Decon 2
  - Decon 3
  - Decon 4

Material

Steel
Aluminum
Glass
Polycarbonate
CARC (Paint)
Viton
Kapton
Silicone



Responses

- Pass/Fail
- Yes/No
- Not Cracked/Cracked
- Safe/**Caution**/Unsafe
- Not Corroded/ **Moderately Corroded**/ Severely Corroded

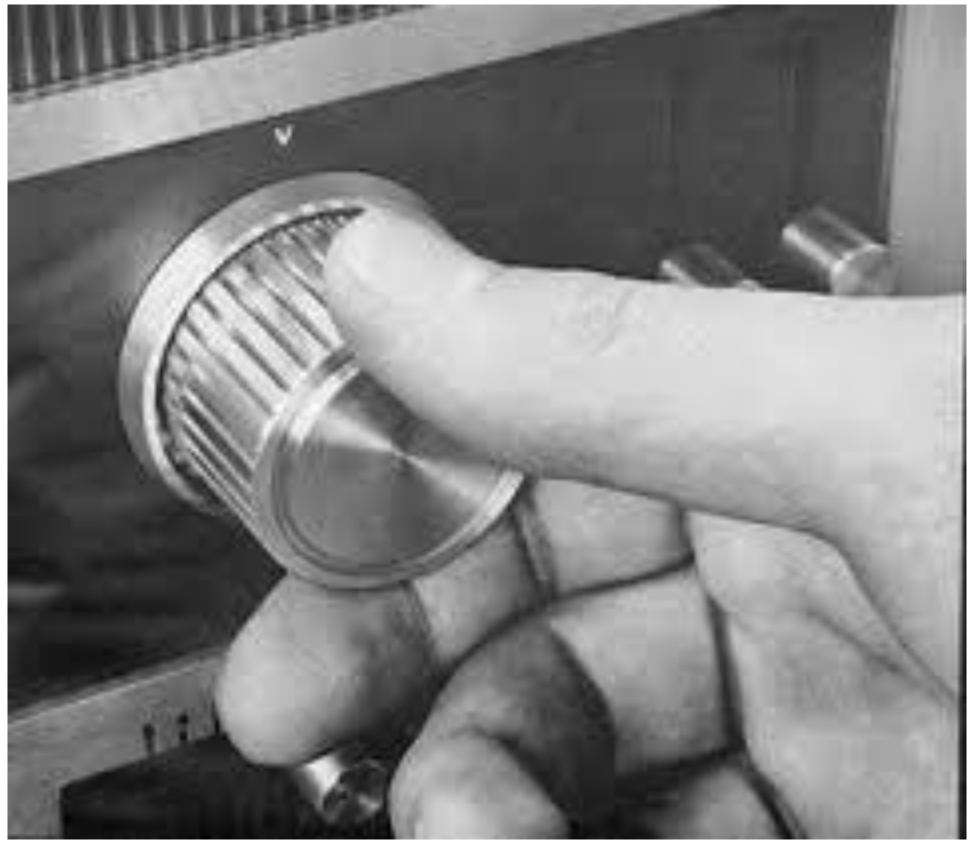
Continuous Factors and Responses

Factors

- Time
- Temperature
- Amount of Agent/Unit Area
- Wind Speed
- Humidity

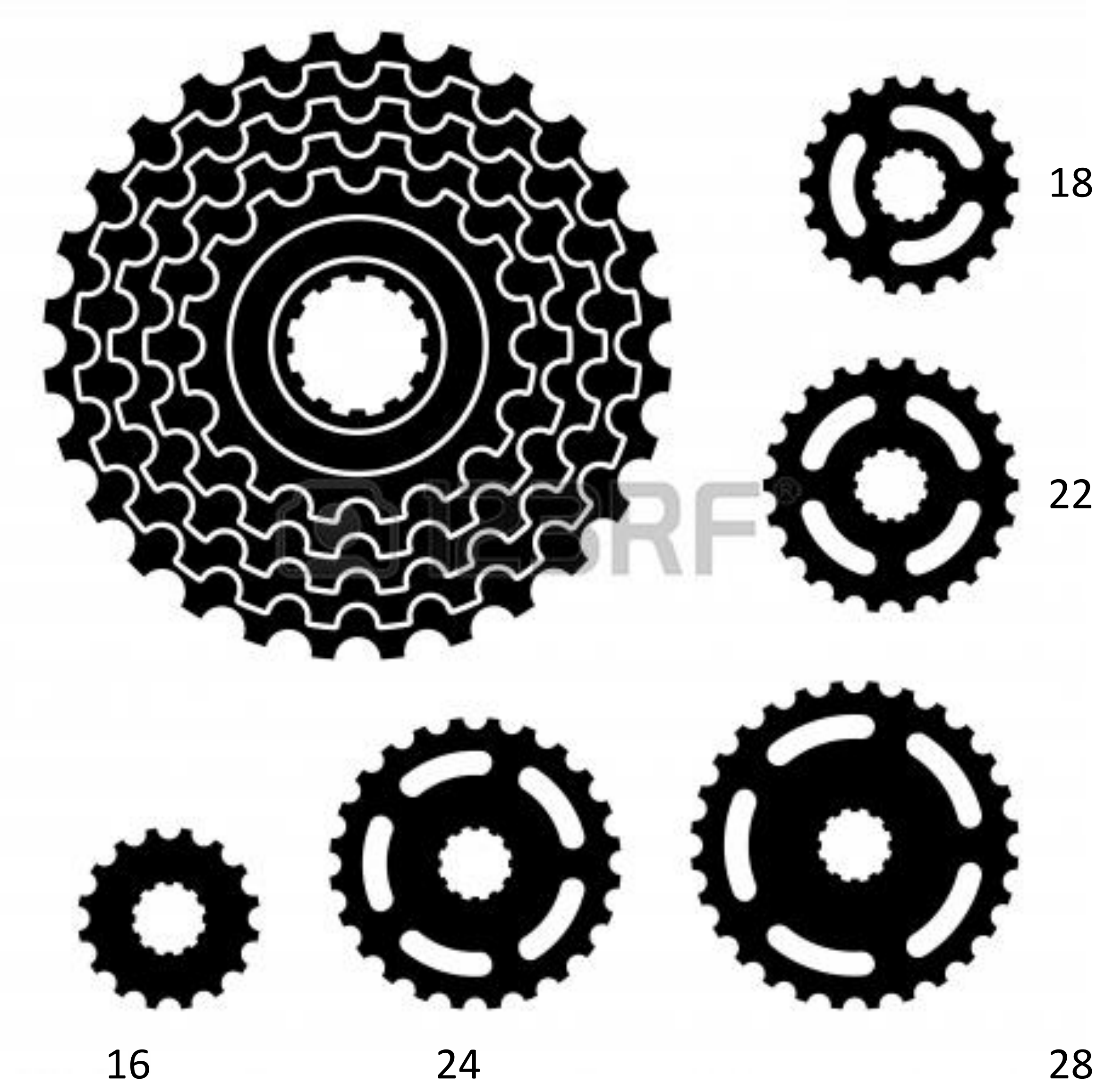
Responses

- Evaporation Rate
- Absorption
- Adsorption
- Residual Concentration



Discrete Numeric Variable

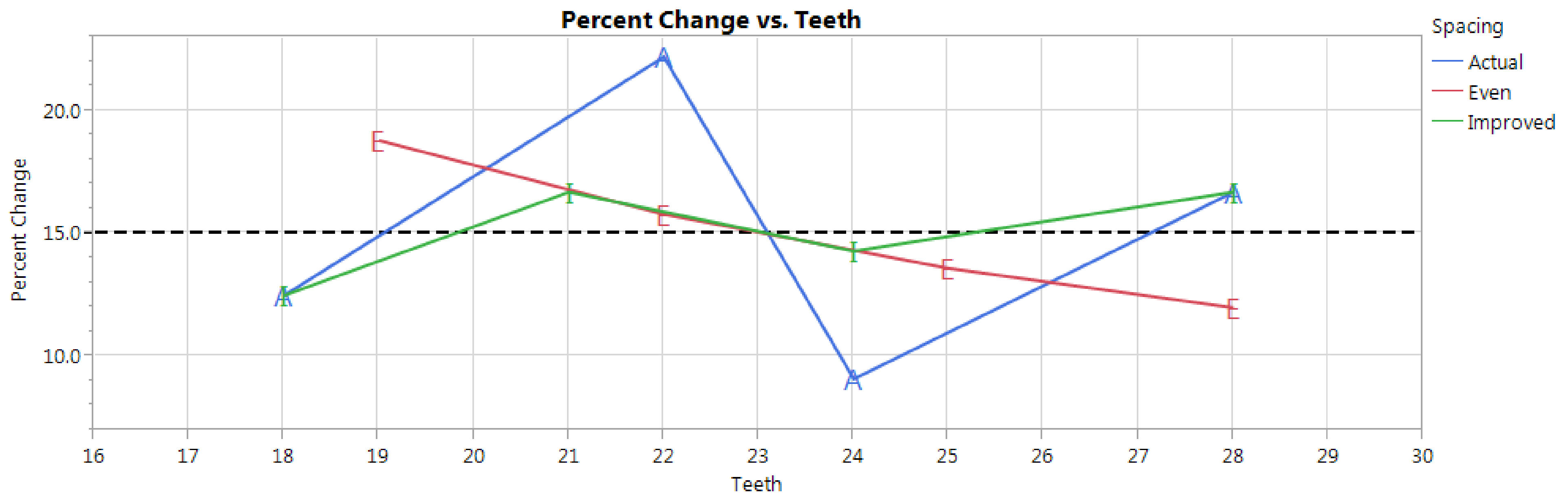
Example: Number of Teeth on Bicycle Sprockets – integer !!!



Evenly Spaced					
Teeth	16	19	22	25	28
Delta		3	3	3	3
% Change		18.8%	15.8%	13.6%	12.0%

Actual Spacing					
Teeth	16	18	22	24	28
Delta		2	4	2	4
% Change		12.5%	22.2%	9.1%	16.7%

Improved Spacing					
Teeth	16	18	21	24	28
Delta		2	3	3	4
% Change		12.5%	16.7%	14.3%	16.7%



Designs like a categorical factor

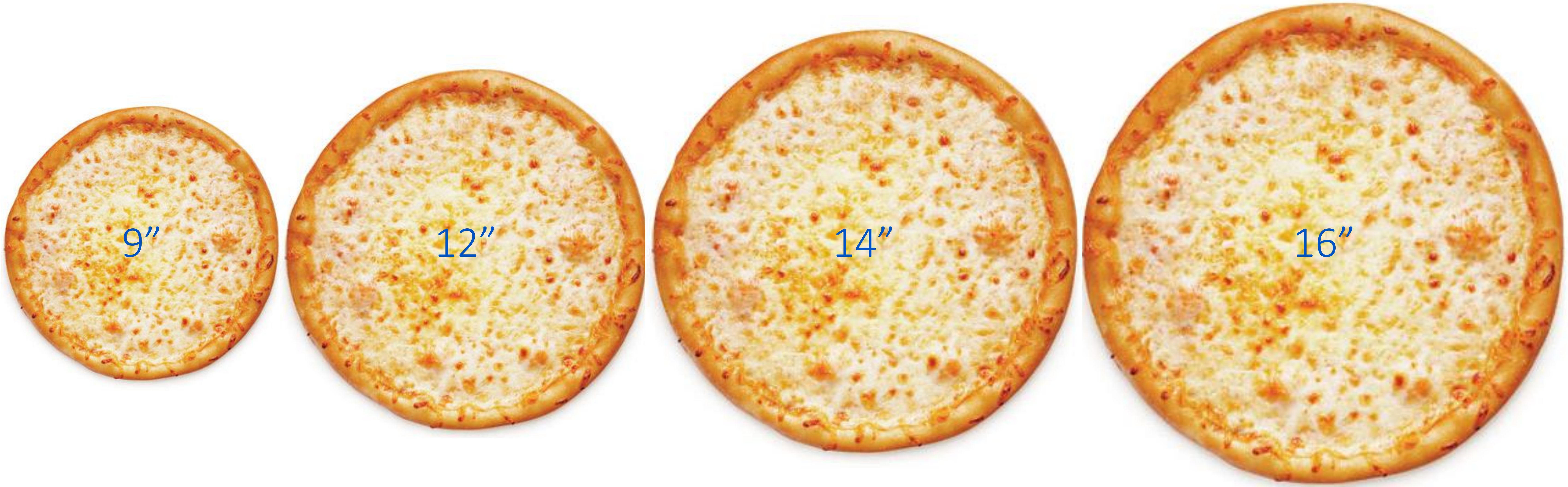
Models like a continuous factor

Discrete Numeric Variable

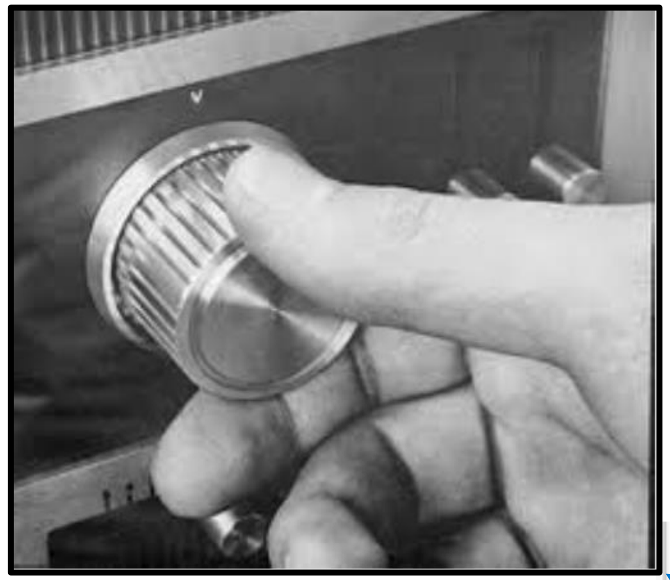
Have only four sizes of pizza pan: 9", 12", 14" & 16" in diameter.
Sizes are not evenly spaced and missing mid-point of full range,12.5".

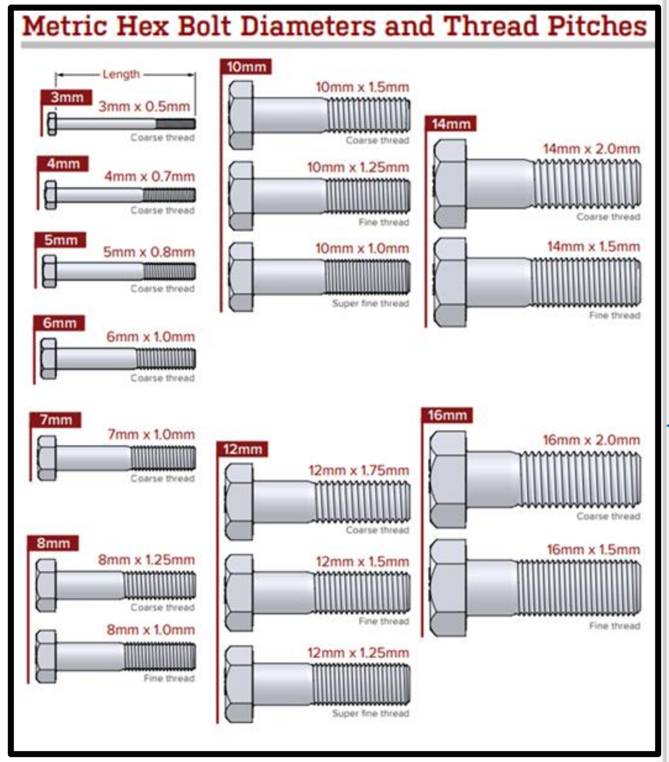
If size treated as <u>continuous</u> factor, 9" to 16" range entered, & model specified as quadratic, then JMP will produce <u>design with mid-points</u> of 12.5".

If size treated as <u>discrete numeric</u> factor, all four sizes entered, & model specified as quadratic, then JMP will produce <u>design with all four levels</u>. There will be more 9s & 16s (extremes), than 12s & 14s (more central).



Three Types of Factors Supported





Factors

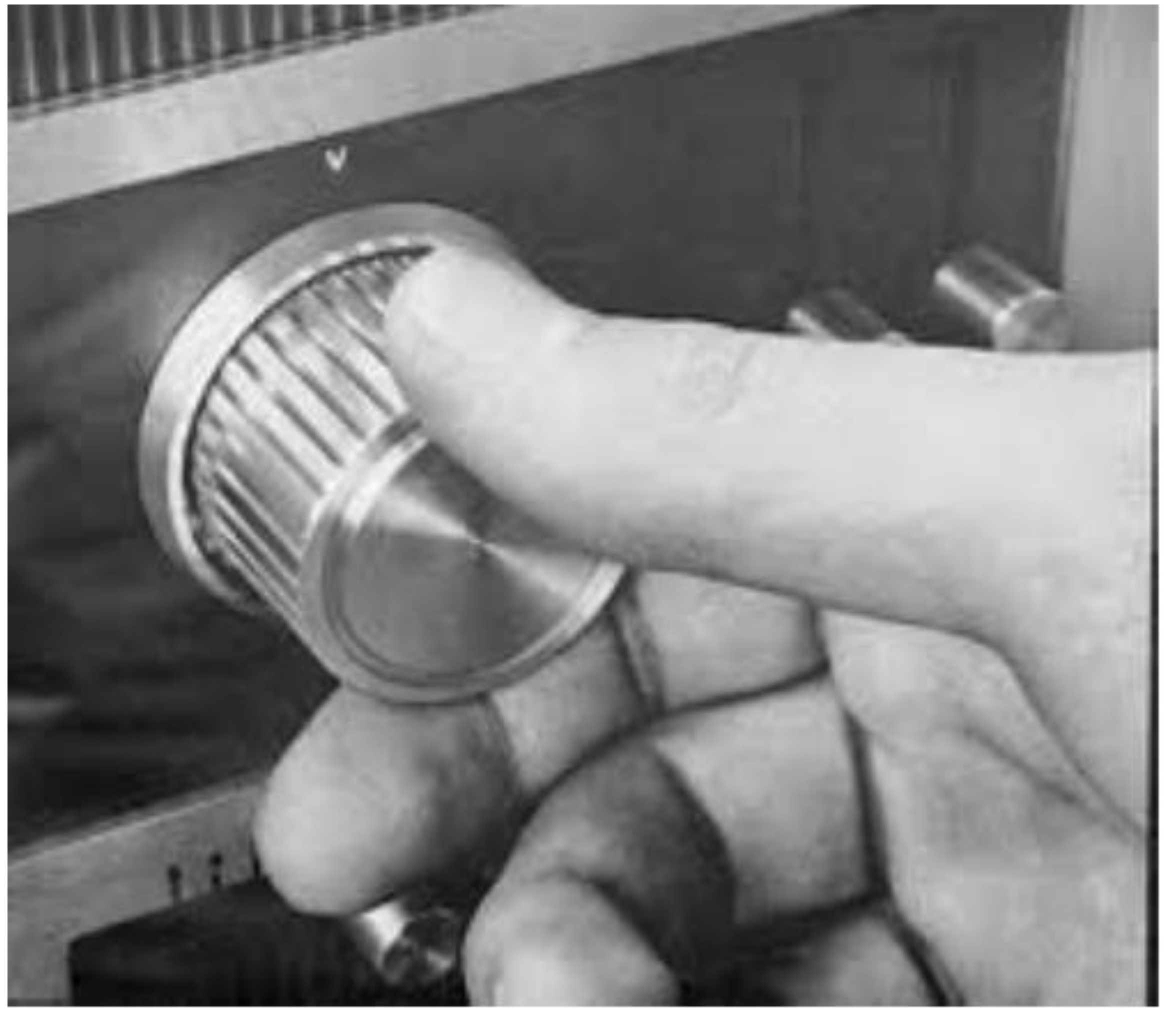
Choices

Role

Add a Continuous Factor — A continuous factor can take any numeric value between a low and a high level.
▶ Show Hint

Add a Discrete Numeric Factor — A discrete numeric factor lies between a low and a high numeric value, but it can be set to user-specified values.
How many levels do you have? 2
▶ Show Hint

Add a Categorical Factor — A categorical factor can take on a specified number of categories, groups, or types.
How many levels do you have? 2
▶ Show Hint

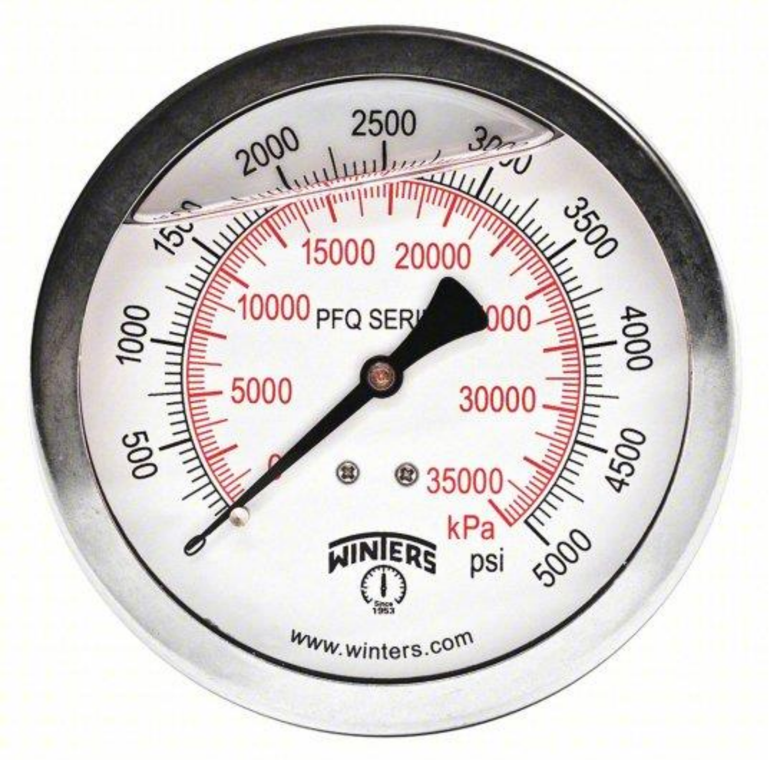


Continuous Factors are finely adjustable over a range.

Think, can I turn a control knob to adjust to any setting?

Examples (Clockwise) are

Time, Temperature, Speed, RPM,* and *Pressure



Categorical Factor: *Vendor*

Order doesn't matter.

Interpolation makes no sense.

L1

L2

L3

Categorical Factor: *Vendor*

Order doesn't matter.

Interpolation makes no sense.

L1 BMB Fasteners, Inc.

L2 CHAMPION BOLT & SUPPLY INC.

L3 AA ANCHOR BOLT, INC.

Categorical Factor: *Vendor*

Order doesn't matter.

Interpolation makes no sense.

L1 BMB Fasteners, Inc.

L2 CHAMPION BOLT & SUPPLY INC.

L3 A ANCHOR BOLT, INC.

Categorical Factor: *Grade of Stainless Steel*

Order potentially matters.

Ordinal Ranking may make sense.

L1 304 Stainless Steel Pros and Cons

The main benefit is that 304 stainless steel is usually considered to be one of the strongest of the mild steels available on the market. It boasts a respectable level of corrosion resistance and is **much easier to mold** than its 316 stainless steel alternative. However, like 18-8 grade stainless steel it is vulnerable to corrosion when exposed to salt water. 304 stainless steel **costs more than 18-8 but less than 316 stainless steel**.

L2 18-8 Stainless Steel Pros and Cons

As already mentioned, 18-8 grade stainless steel is celebrated for its superior level of corrosion resistance. However, it is known to show signs of corrosion when exposed to chlorides, such as salt. Therefore, it is not the ideal stainless steel to use for marine applications. On the upside, 18-8 grade stainless steel properties include the fact that **it can be bent and molded** without it having an effect on its overall strength and durability. This type of stainless steel is also not only **extremely budget-friendly**, but it also requires little to no maintenance. 18-8 stainless steel yield strength is also impressive.

L3 316 Stainless Steel Pros and Cons

316 stainless steel boasts a higher strength and durability than 304 stainless steel. It also has a higher level of corrosion resistance, including when exposed to salt water. It performs well against pitting and is also resistant to caustic chemicals. As mentioned above, however, 316 stainless steel is **less malleable** than 304 stainless steel. It is also **substantially more expensive.**

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Categorical Factor: *Vendor*

Order doesn't matter.

Interpolation makes no sense.

L1 BMB Fasteners, Inc.

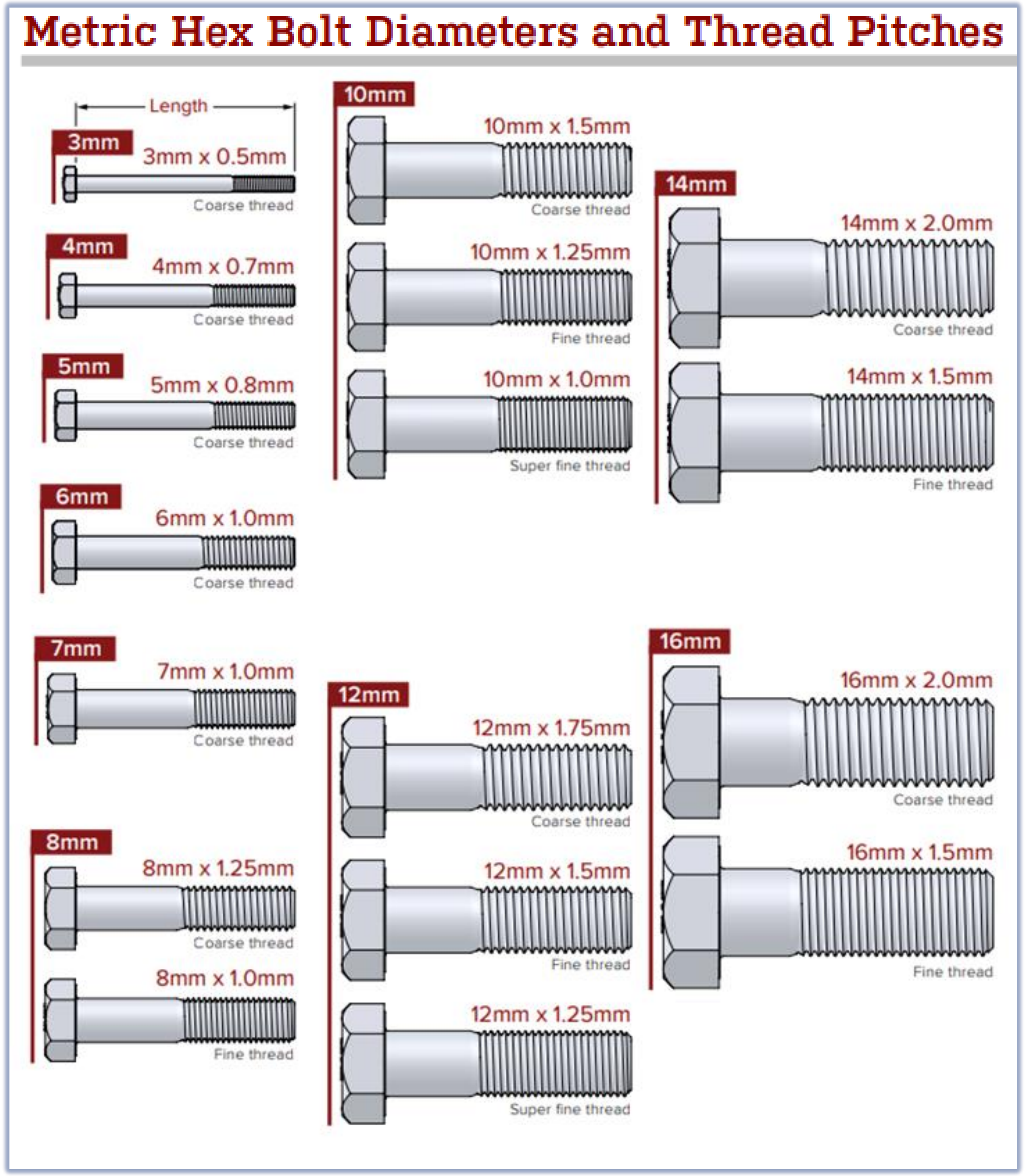
L2 CHAMPION BOLT & SUPPLY INC.

L3 A ANCHOR BOLT, INC.

Discrete Numeric Factor: *Diameter*

Order does matter.

Interpolation makes sense.



Designs like a *categorical* factor, but models as *continuous*

Bolt diameters are only available in whole millimeters between 3 & 16, with no choice of 9, 11, 13, & 15 mm.

For range of 7 to 10, mid point is ~~8.5~~. Only "mid" level is 8 mm which is unevenly spaced between ends.

For range of 10 to 16, mid point is ~~13~~. Only "mid" levels are evenly spaced, 12 & 14 mm.

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Mixture Variables

Simple mixture – Making salad dressing

- Relative ***proportions*** of factors or components is more important than actual quantity
- Three liquid components - Oil, Water, and Vinegar
- 8 oz. in Cruet vs. 4 gal. in Jug

5 oz.	“O”	320 oz.	5/8
1 oz.	“W”	64 oz.	1/8
2 oz.	“V”	128 oz.	1/4

- To study these mixture components in a DOE use ranges that are proportions:

O:	0.500 to 0.750	(½ to ¾)
W:	0.000 to 0.250	(0 to ¼)
V:	0.125 to 0.375	(⅛ to ⅜)

- Sum of proportions ***constrained*** to equal 1.

100.0%

37.5%

25.0%

0%

1 = O + W + V so therefore…

W = 1 – (O + V), O = 1 – (V + W), & V = 1 – (O + W)

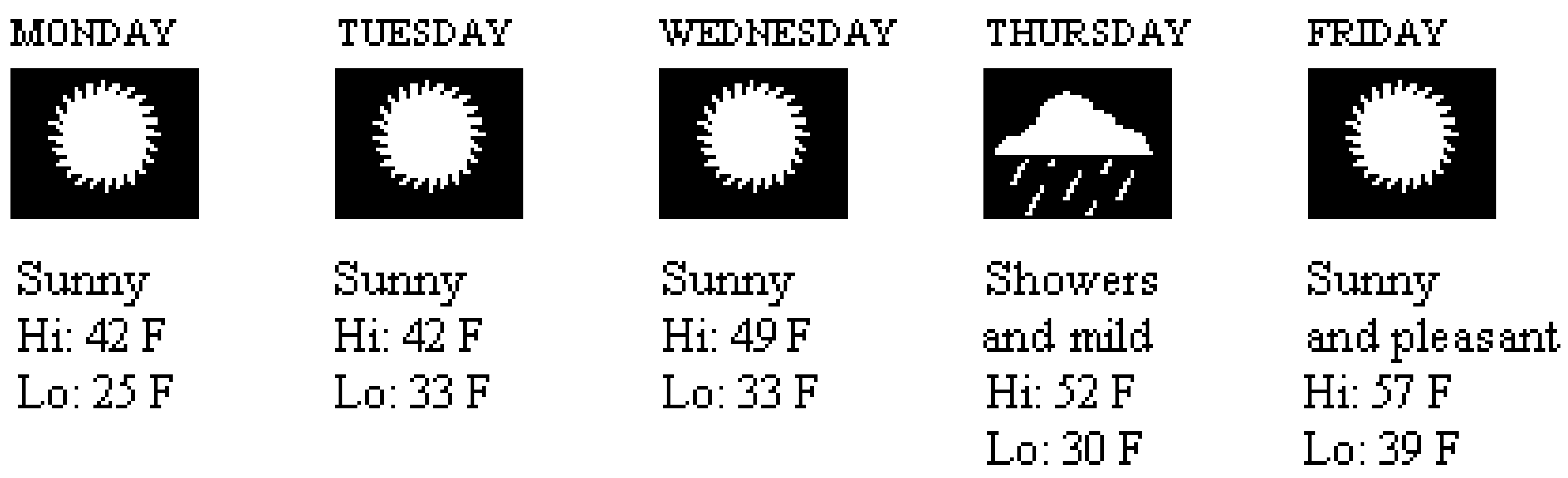
DOE Recordings at www.jmp.com/fedgov

These 12 videos primarily cover Design of Experiments (DOE) topics.

Custom DOE – Make the Design Fit Your Problem (Link to Mastering JMP)	Screening Designs Classic FF & PB, and Modern D-Optimal, Supersaturated, DSD, & Alias-Optimal	Compare Designs How to Choose Better Designs on Multiple Criteria
Advanced Custom DOE - Augmentation, Broken Design Repair, & Design from a Candidate Set	Definitive Screening Designs (DSD) Creation & Augmentation	Data Transformations Get Rid of L-o-F, Predictions Make Physical Sense (Link to Mastering JMP)
Mixture DOE Efficiently Blending Ingredients to Optimize a Process (Link to Mastering JMP)	Analyzing DSD DOEs Graphical Methods and Fit Definitive Screening	Power Calculation via MC Simulation Binary Responses & Split-Plot Designs
Efficient M&S Using DOE How to Run Fewer Computer Simulations	Exploratory Data and Root Cause Analyses What to Do When You Don't Have a DOE	Covering Arrays - Rapid Fault Detection in Software & Systems

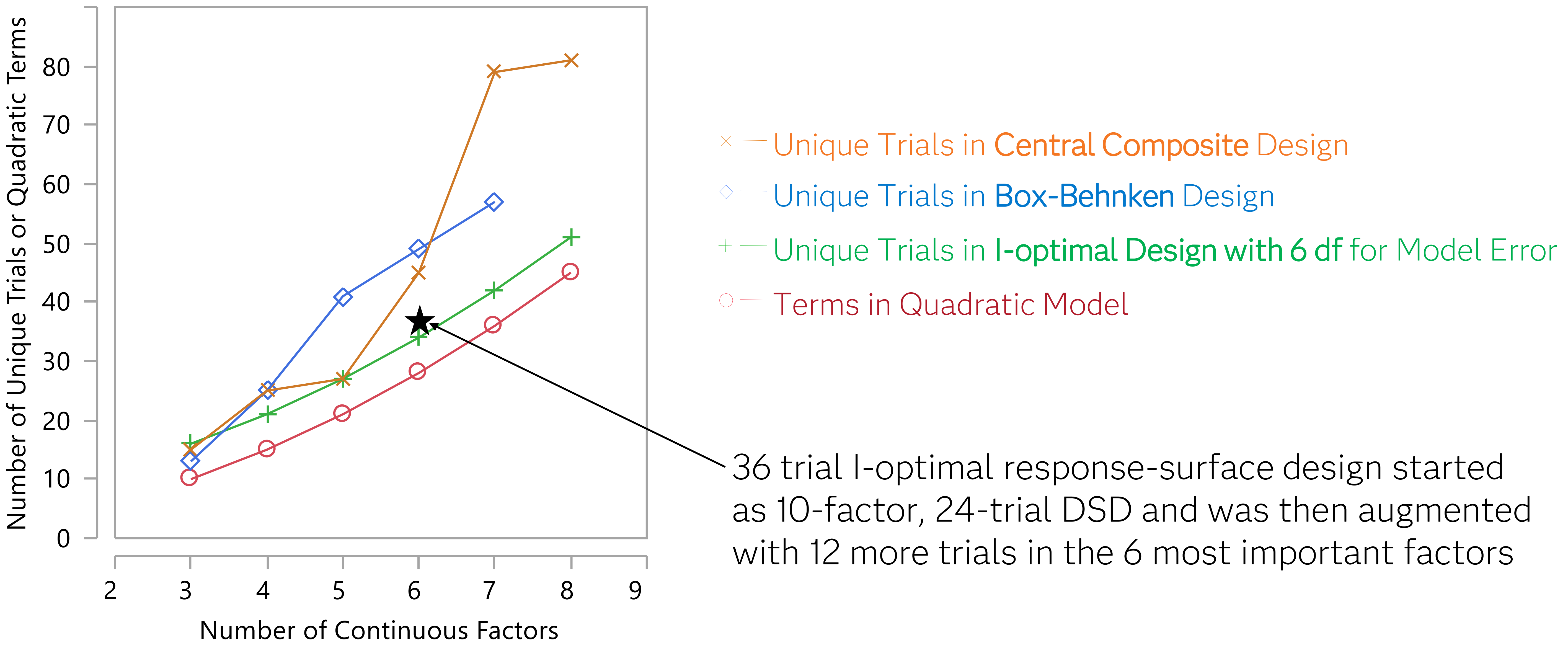
https://community.jmp.com/t5/US-Federal-Government-JMP-Users/Mixture-DOE-JMP-14/ta-p/69546

Blocking Factor like "Day" or "Batch"



- A design run over 5 days that is sensitive to humidity might SHIFT on Thursday
  - But what if because of the rain the tester from days 1, 2, 3 & 5 didn't make it to work?
  - What if that day the power went out briefly? Or all-hands meeting "paused" the work? Or…?
- The block variable doesn't tell you the cause of the effect - just that a shift has been detected among blocks.
- Hoping block variable has no effect. If it does, then how can we reliably predict other blocks? If significant, it probably means we are missing a factor.
- The only way to be sure that no "unknown" factor has crept into the experiment, is to test for it - and "blocking" your design is an inexpensive insurance policy to buy.
- Block variable is a categorical factor having only 1-way effects (no interactions)

Number of Unique Trials for 3 Response-Surface Designs and Number of Quadratic Model Terms vs. Number of Continuous Factors

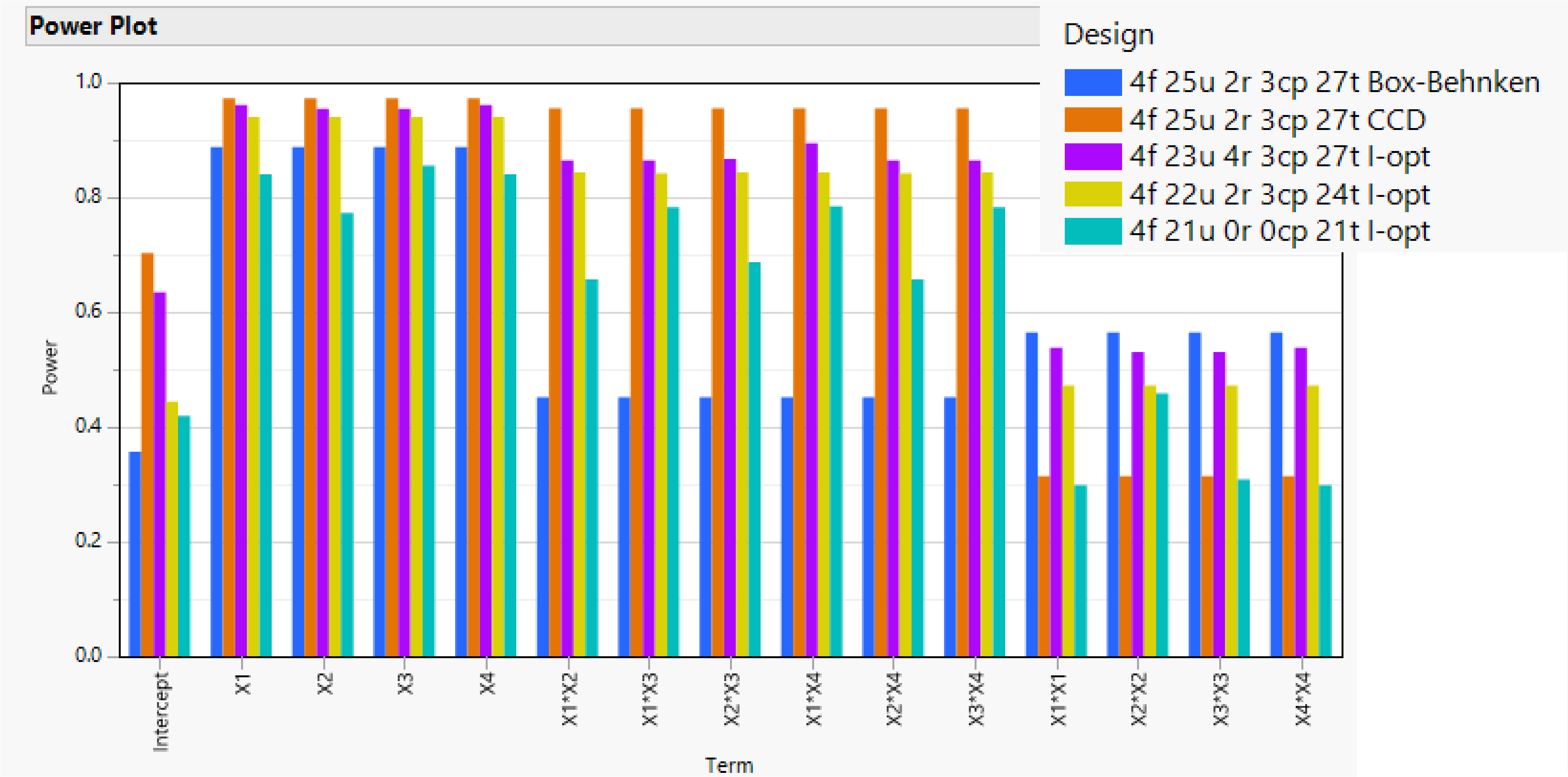


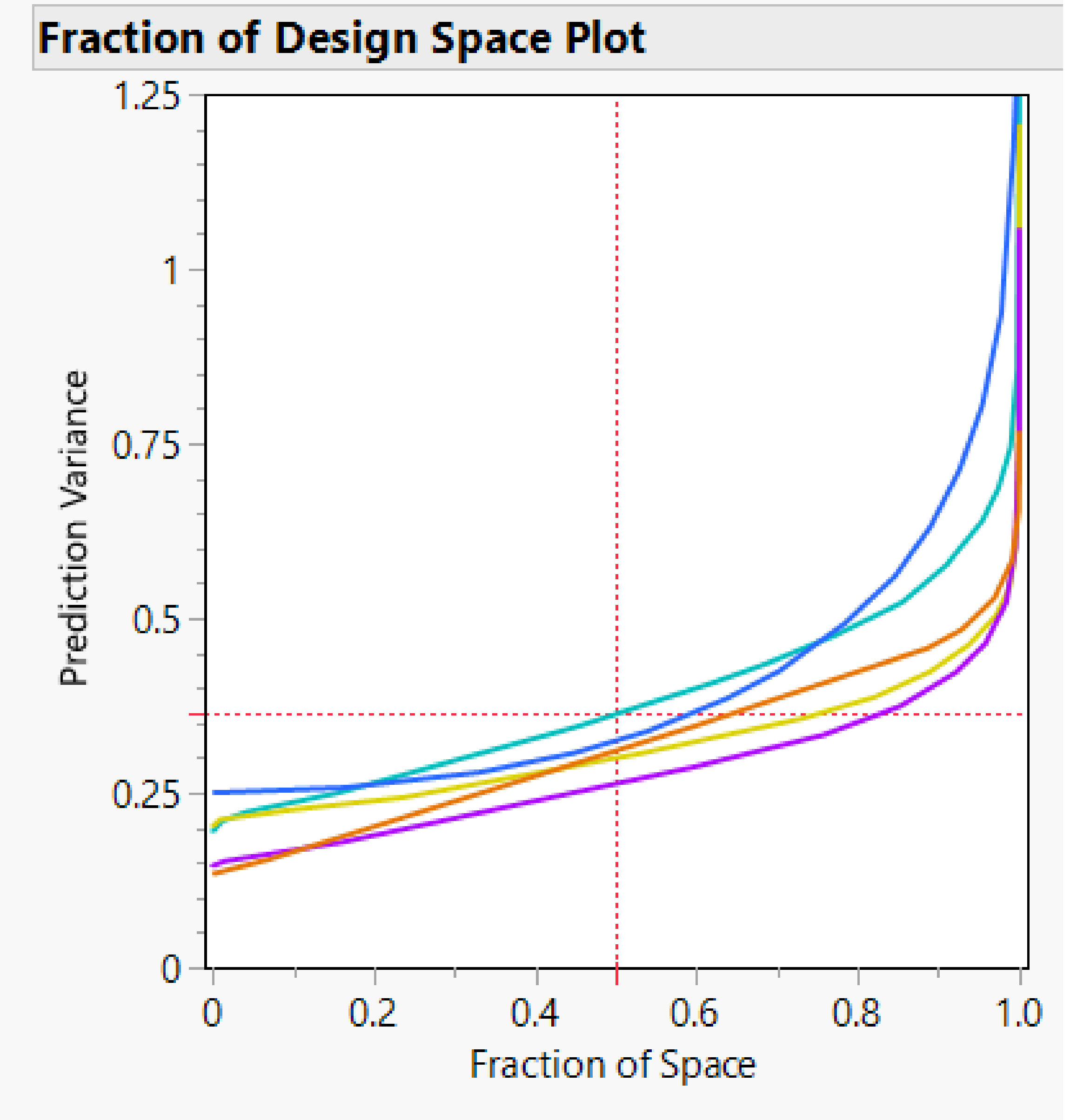
If generally running 3, 4 or 5-factor fractional-factorial designs…

1. How many interactions are you not investigating?
2. How many more trials needed to fit curvature?
3. Consider two stages: Definitive Screening + Augmentation

Power & Fraction of Design Space Plots

Comparison for same sized, 27-trial 4-factor designs: Box-Behnken, Central Composite, I-optimal, AND 24-trial & 21-trial I-optimal designs





BB best for Quadratics
CC best for Main Effects & Interactions
IO-27 strong second for ALL
IO-24 nearly as good

BB highest Prediction Variance
CC lower and flatter than BB
IO-27 lowest & flattest Prediction Variance
IO-24 nearly as good

Recording

 https://community.jmp.com/t5/JMP-On-Air/Can-You-Stop-Using-Classic-RSM-Designs-Cold-Turkey-or-Take-Two-I/ta-p/263202

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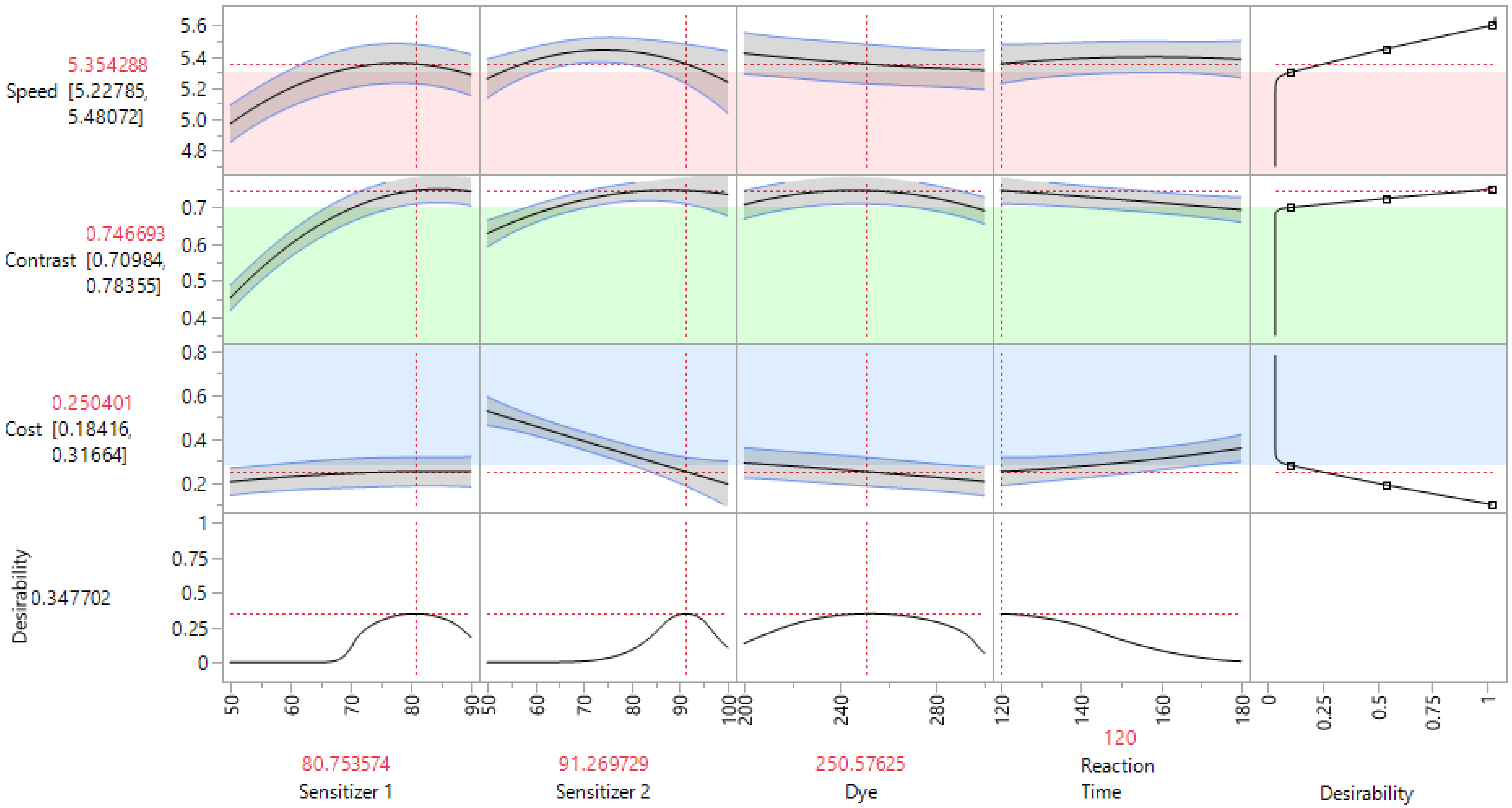
Three Response, Four Continuous Factor, RSM Design

Make the DOE for this analysis

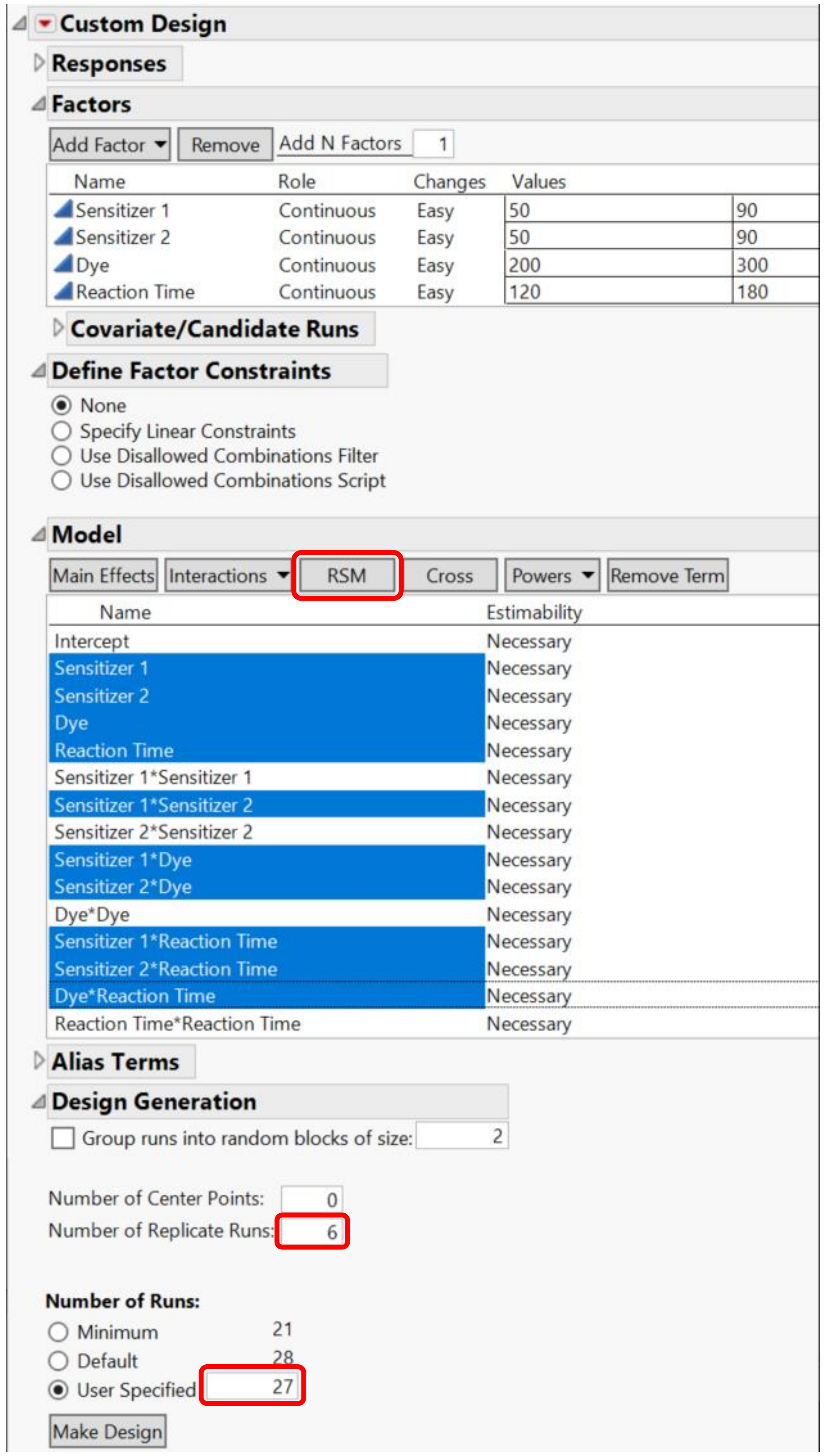


3 Responses & 4 Factors

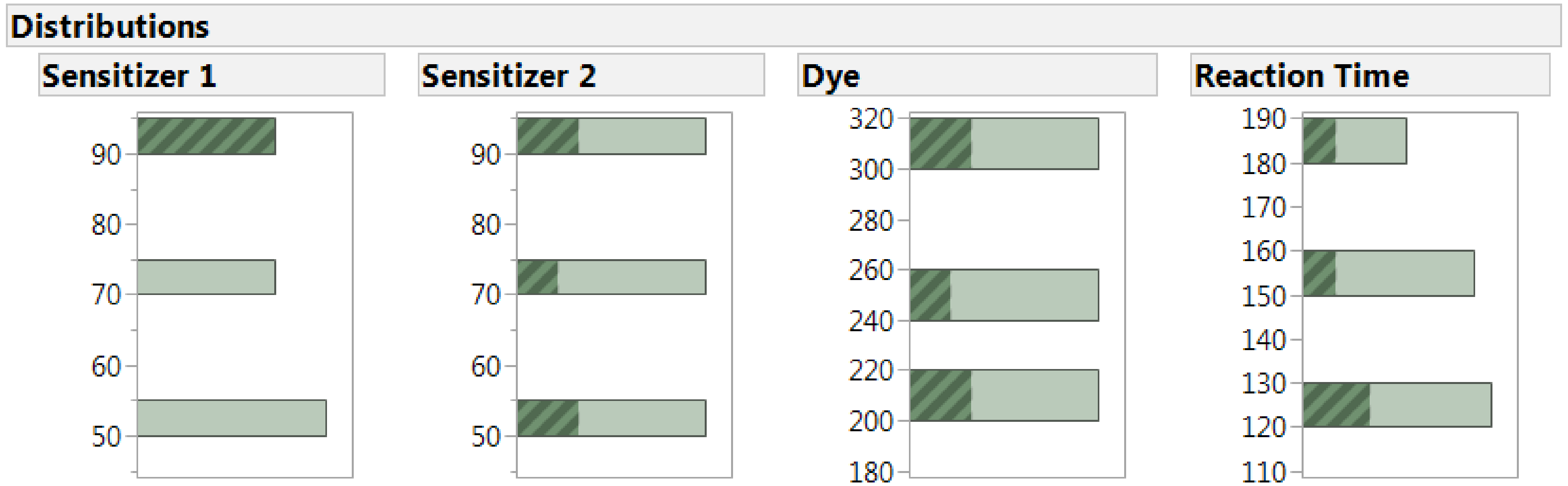
Response	Goal	Requirement
Speed	Maximize	≥ 5.3
Contrast	Maximize	≥ 0.7
Cost	Minimize	≤ 0.28

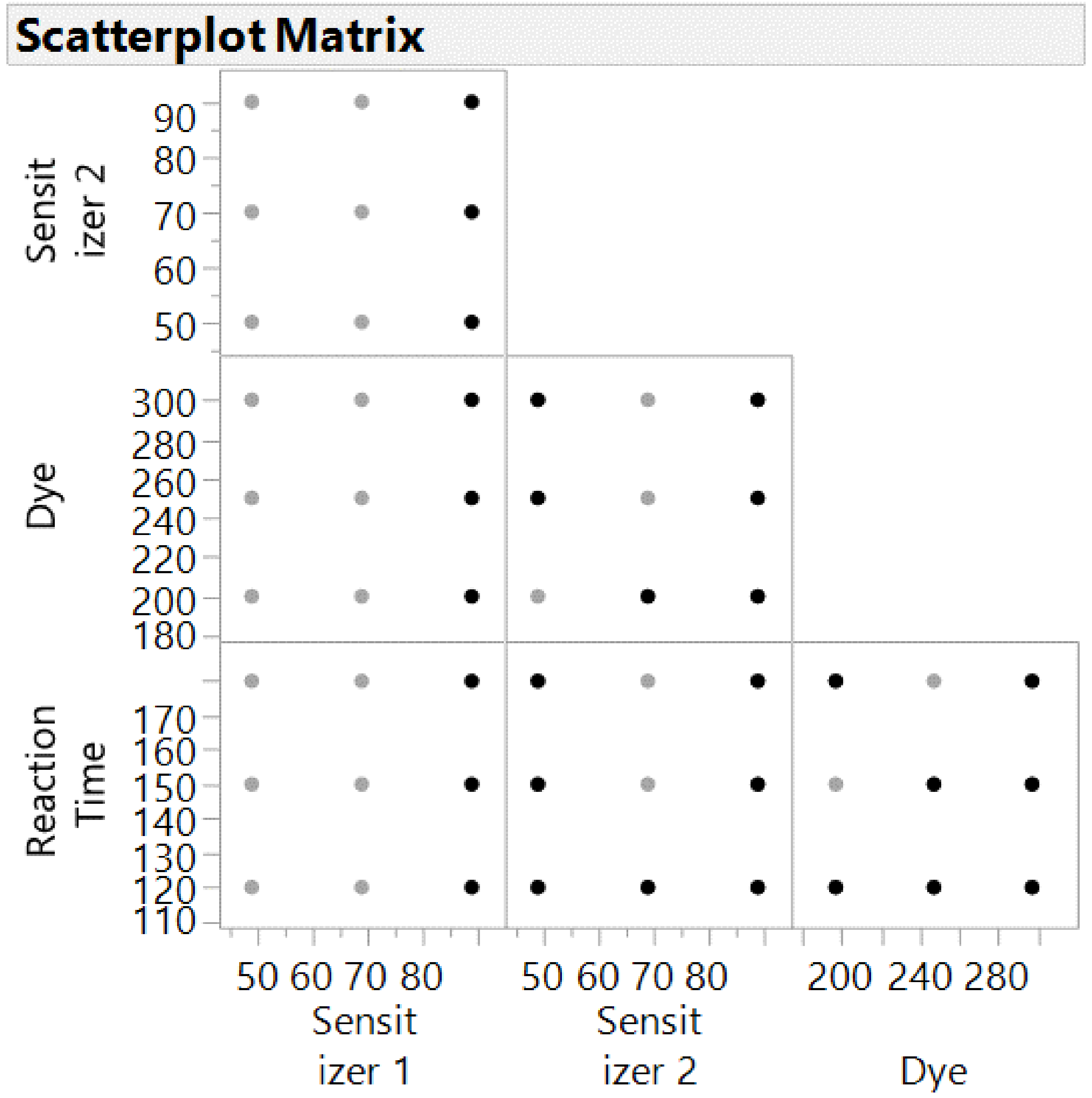
Factor	Lo Limit	Hi Limit
Sensitizer 1	50	90
Sensitizer 2	50	90
Dye	200	300
Reaction Time	120	180

Custom DOE Dialog, Design, Distribution of Design Trials, & Projections of Design Trials in 2-D



	Sensitizer 1	Sensitizer 2	Dye	Reaction Time	Speed	Contrast	Cost
1	50	50	250	120	5.36	0.616	0.198
2	50	50	200	180	5.39	0.537	0.175
3	90	70	200	120	5.31	0.623	0.447
4	50	90	200	150	5.13	0.431	0.177
5	70	70	250	180	5.37	0.643	0.445
6	50	90	300	120	4.79	0.375	0.231
7	90	90	200	180	5.45	0.626	0.471
8	90	50	250	150	5.00	0.470	0.670
9	50	50	300	150	5.22	0.478	0.283
10	70	90	200	120	5.41	0.668	0.226
11	90	90	250	120	5.33	0.734	0.310
12	50	50	250	120	5.32	0.574	0.257
13	70	50	200	150	5.49	0.596	0.456
14	50	70	250	180	5.22	0.558	0.166
15	70	70	250	150	5.57	0.689	0.390
16	90	90	300	150	5.26	0.653	0.226
17	70	70	250	150	5.47	0.688	0.356
18	70	70	300	120	5.42	0.657	0.337
19	50	70	200	120	5.43	0.518	0.222
20	50	50	300	150	5.15	0.505	0.287
21	90	70	200	120	5.33	0.661	0.457
22	50	90	300	120	4.97	0.411	0.191
23	90	50	300	120	5.09	0.492	0.588
24	90	50	300	180	5.03	0.358	0.733
25	70	70	250	150	5.59	0.707	0.318
26	70	90	300	180	5.25	0.605	0.290
27	50	90	200	150	5.24	0.476	0.177





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Distributions of Responses and Factors

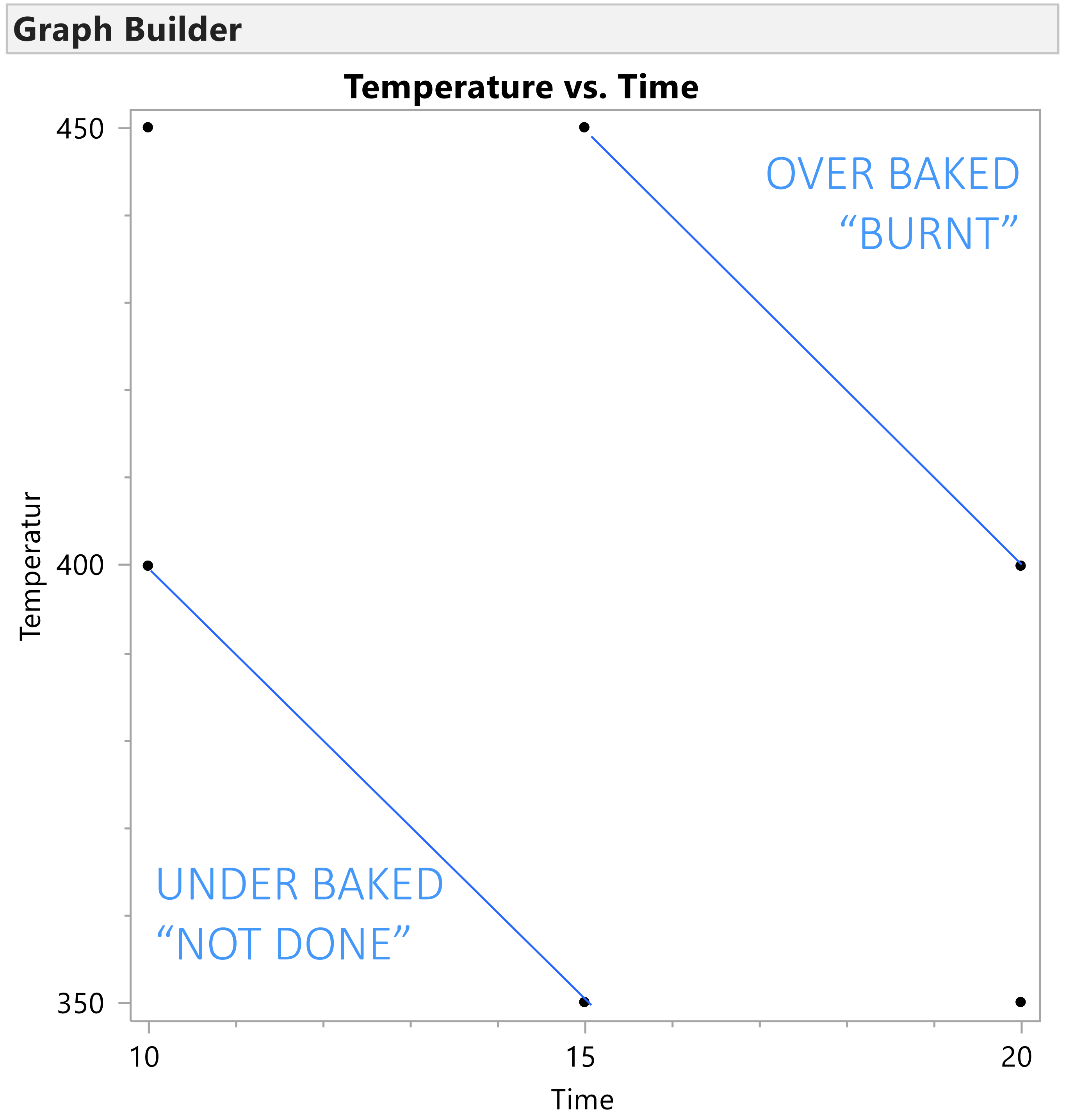
Find Observation with Highest Desirability

4 Factors, 3 Types, 1 Hard-to-Change, Plus 2 constraints

Create DOE for a Real-World Pizza Process

Continuous	• Time:	10 20 (easy)
Continuous	• Temp:	350 450 (hard)
Discrete Numeric with 4 levels	• Pizza Size:	9, 12, 14, & 16 (easy)
Categorical with 3 levels	• Pizza Type:	(easy)

- Cheese
- Meats
- Veggies

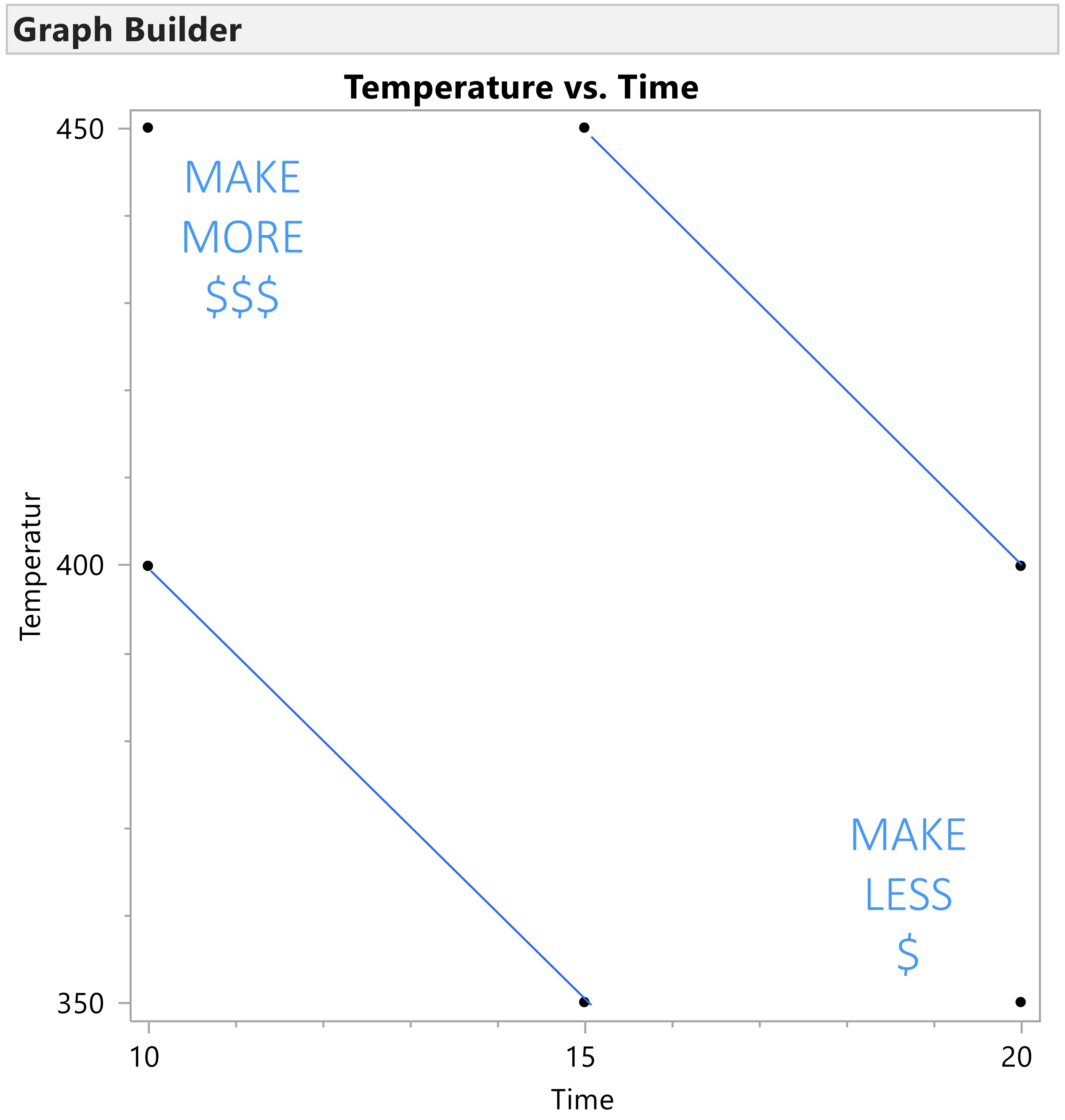


Hi + Hi = "Burnt"

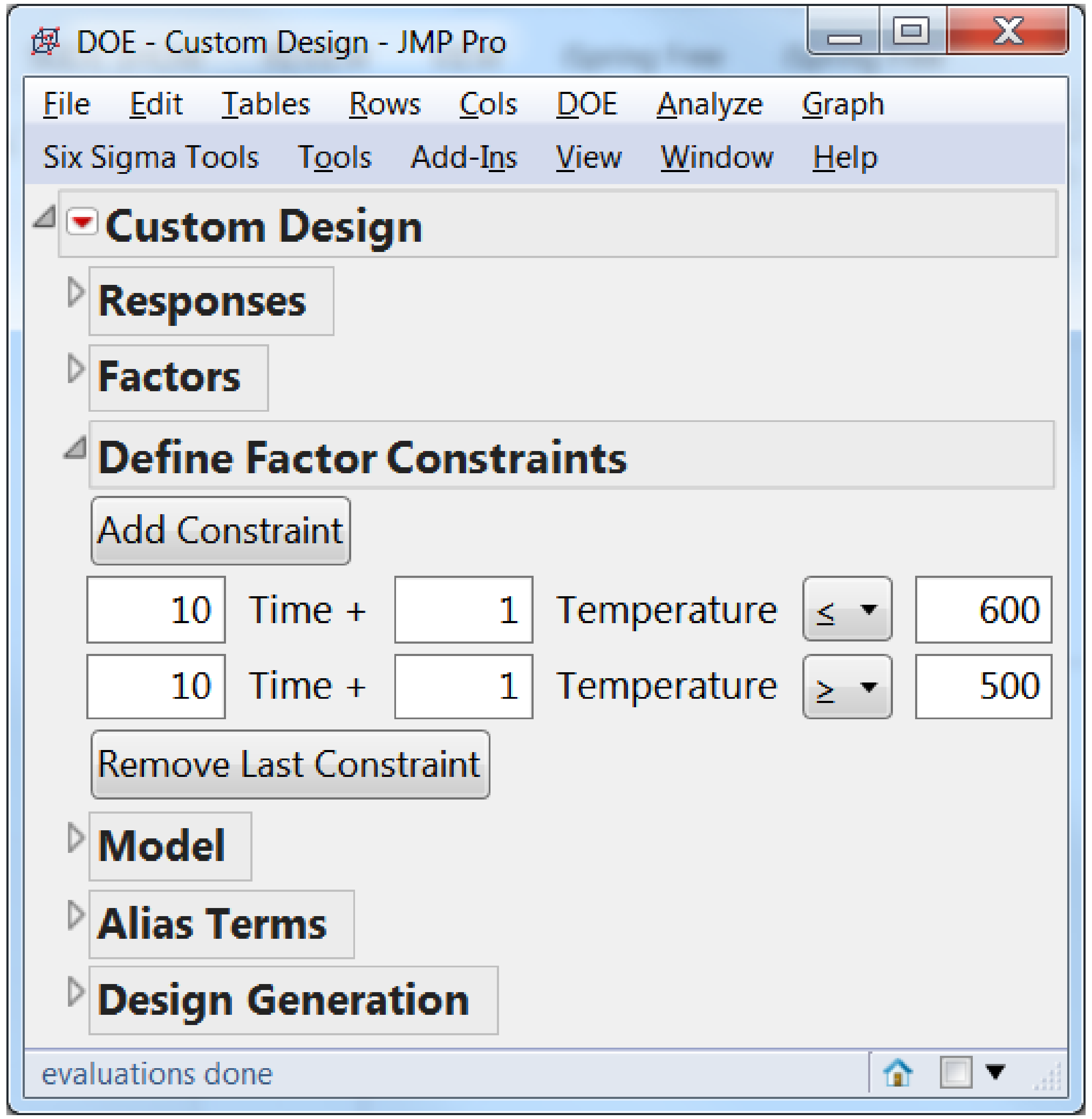
Lo + Lo = "Not Done"

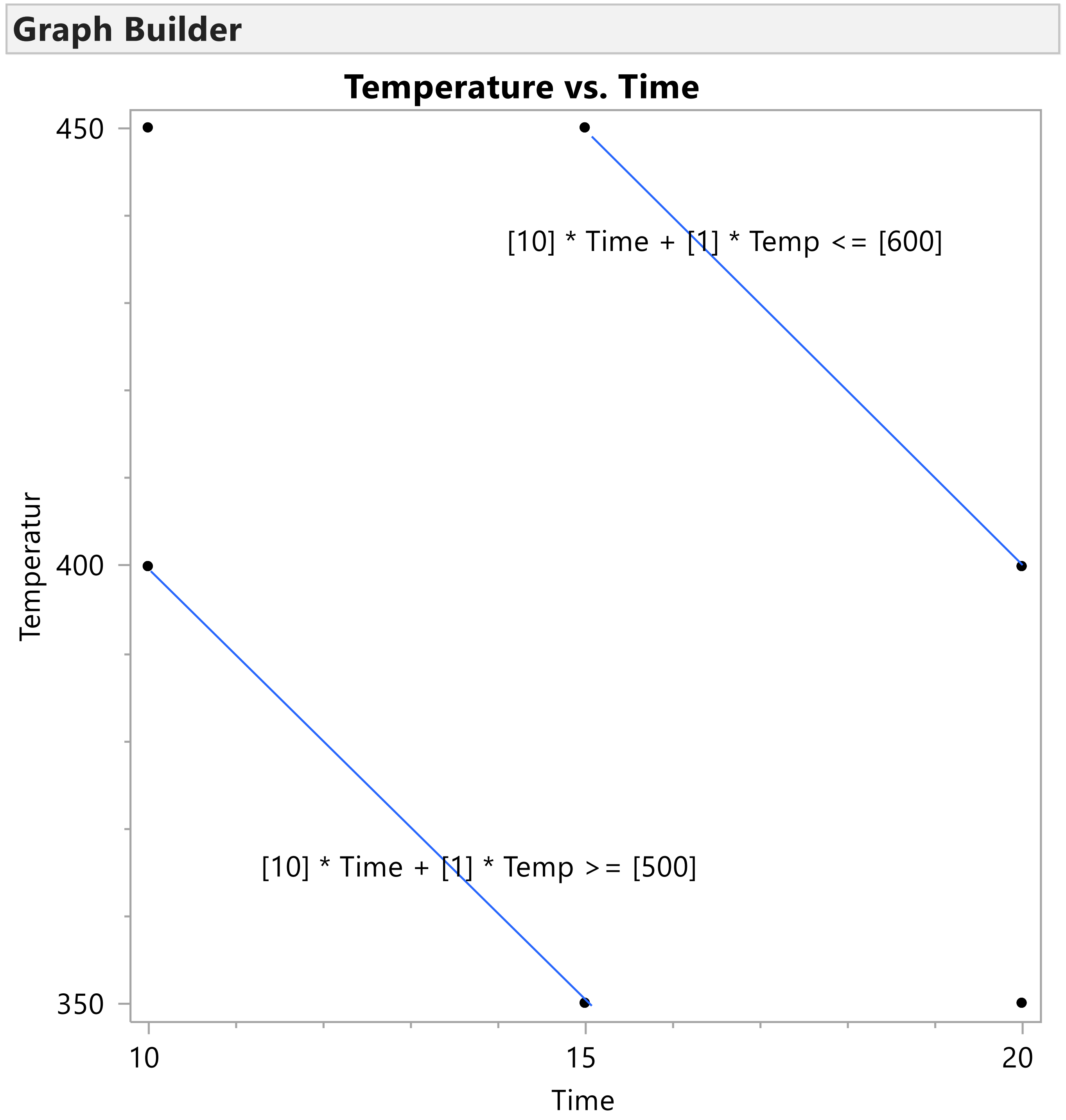
Time and Temperature Constraints

- Shorter times means more pizzas produced per hour
- Make most of your money in a few hours each evening
- *"No pizza shall take more than 7 minutes!"* – Mgmt.



Time and Temperature Constraints Uncoded = Raw Units

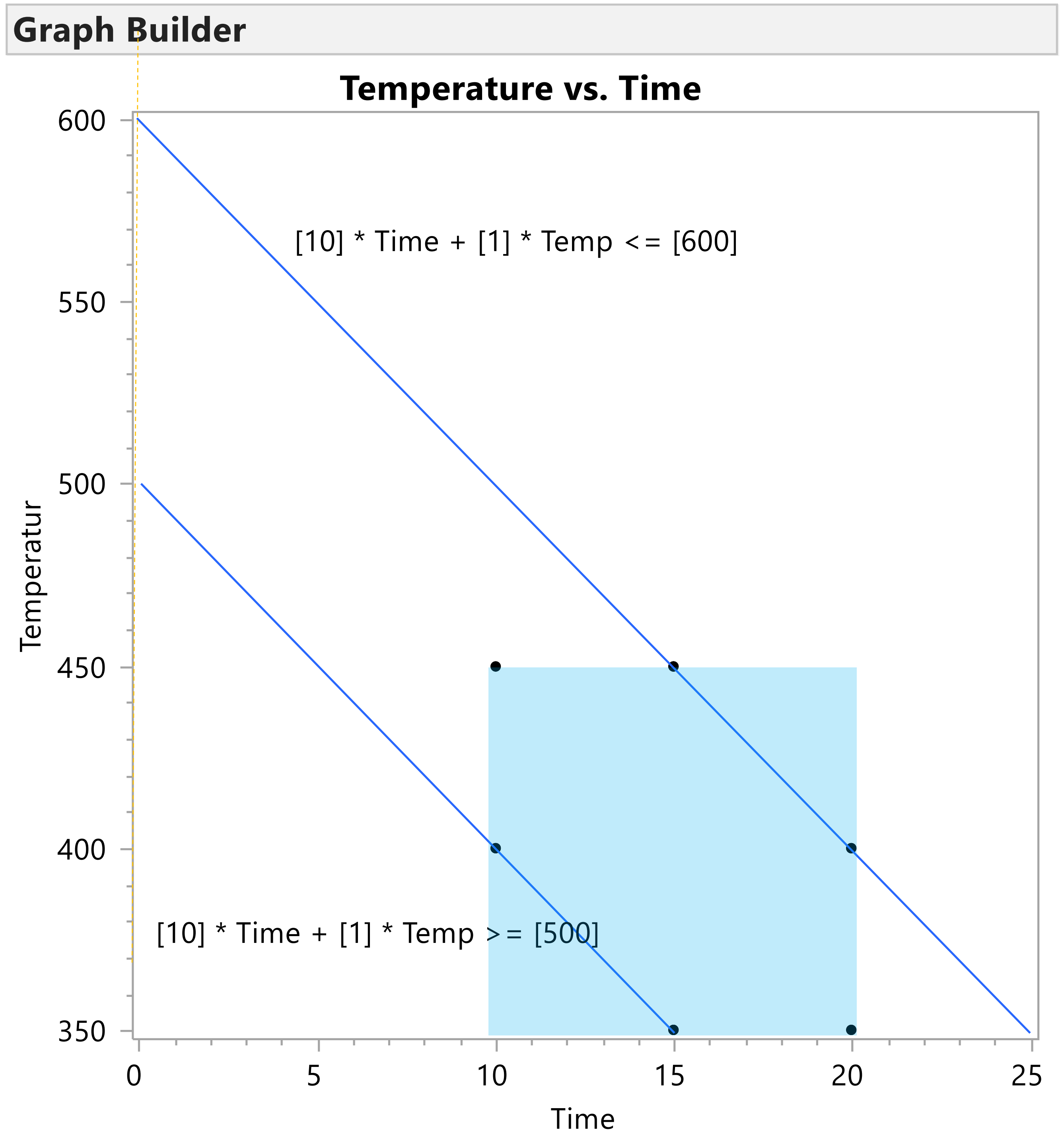


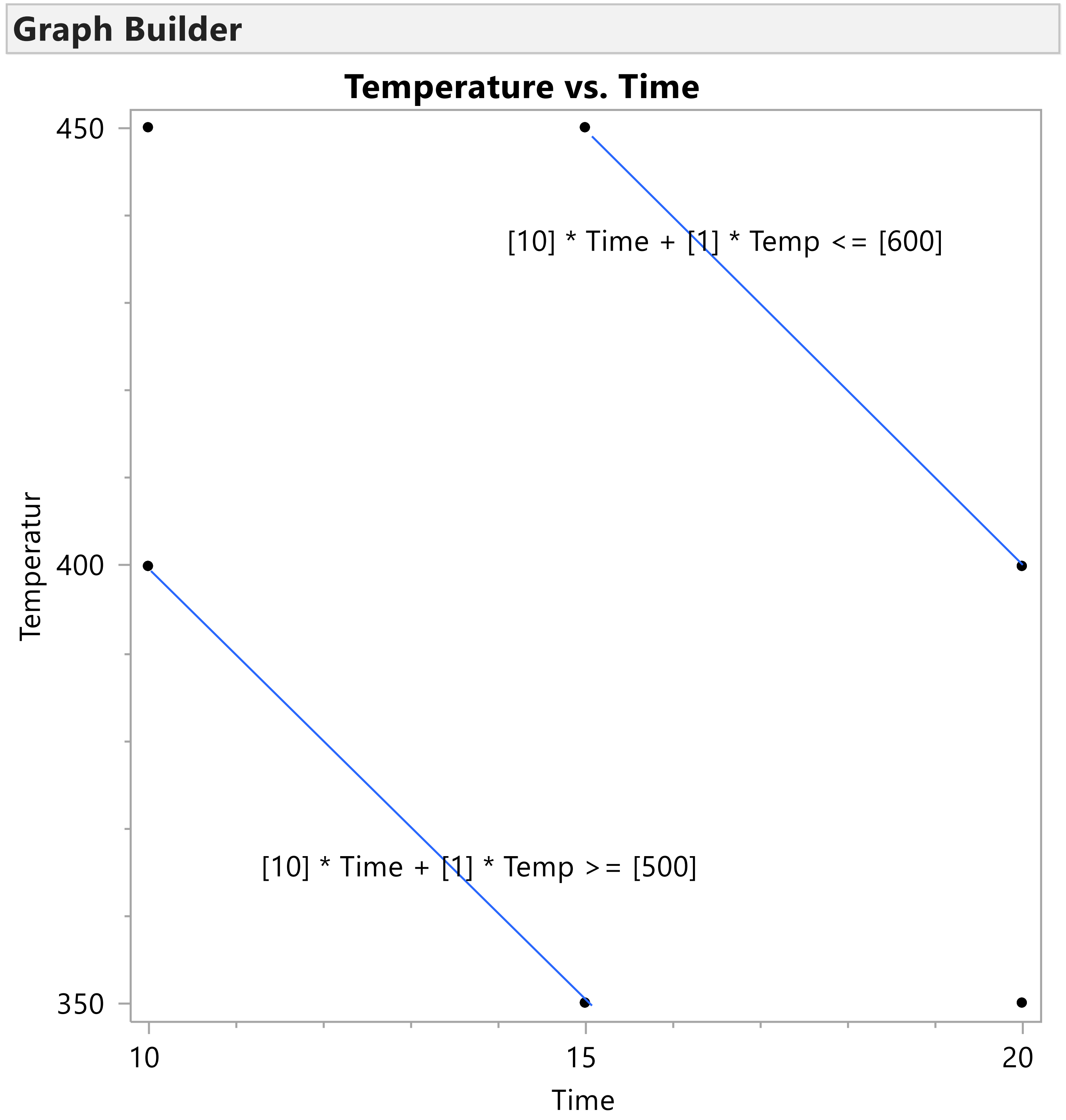


Time and Temperature Constraints
Uncoded = Raw Units

Recall equation of a straight line?

$y = mx + b$





Slope = m = rise/run = -150/15; m = -10
Intercept = b = y when x = zero; b = 600
Intercept = b = y when x = zero; b = 500

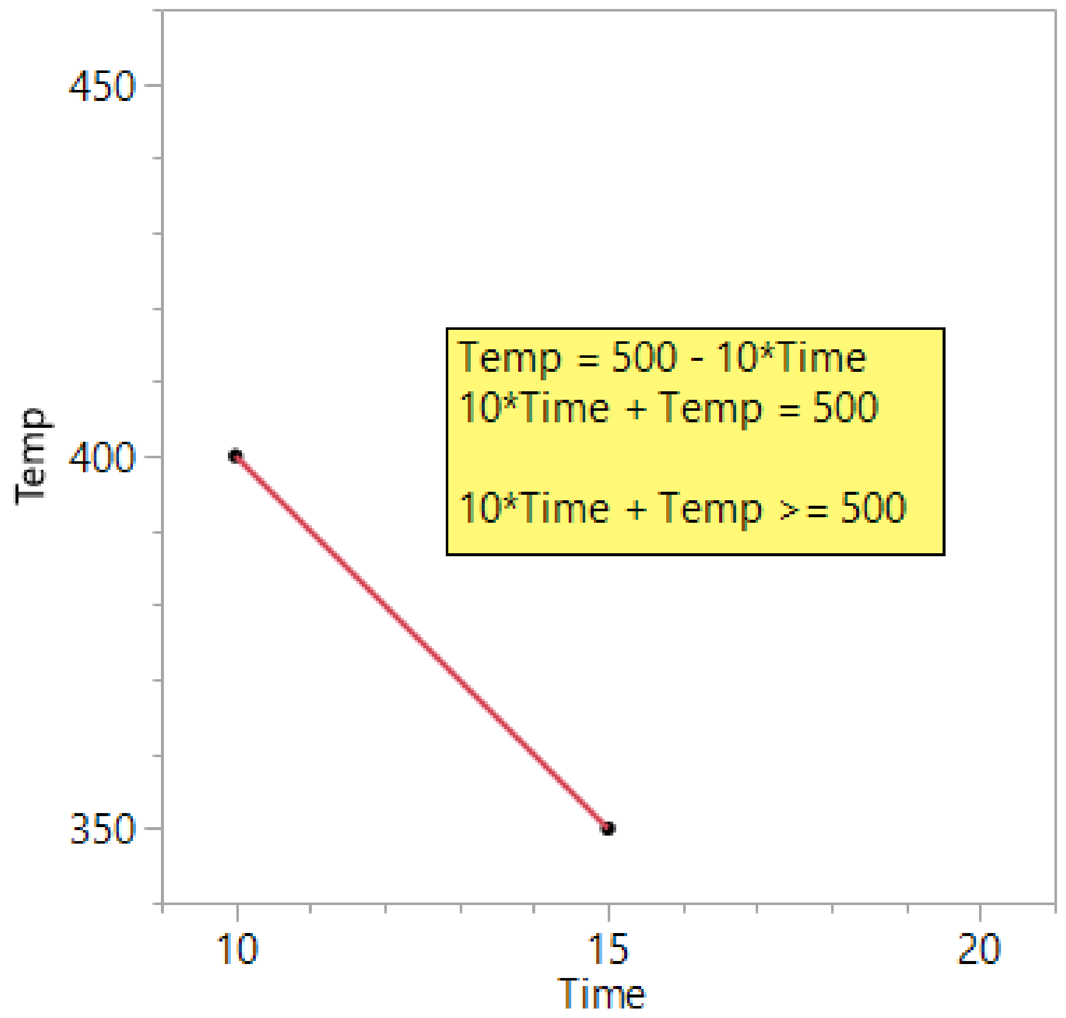
Time and Temperature Constraints Uncoded = Raw Units

	Time	Temp	Constraint Location
1	15	450	Upper
2	20	400	Upper
3	15	350	Lower
4	10	400	Lower

Have JMP solve

$y = mx + b$

Bivariate Fit of Temp By Time Constraint Location=Lower



Linear Fit

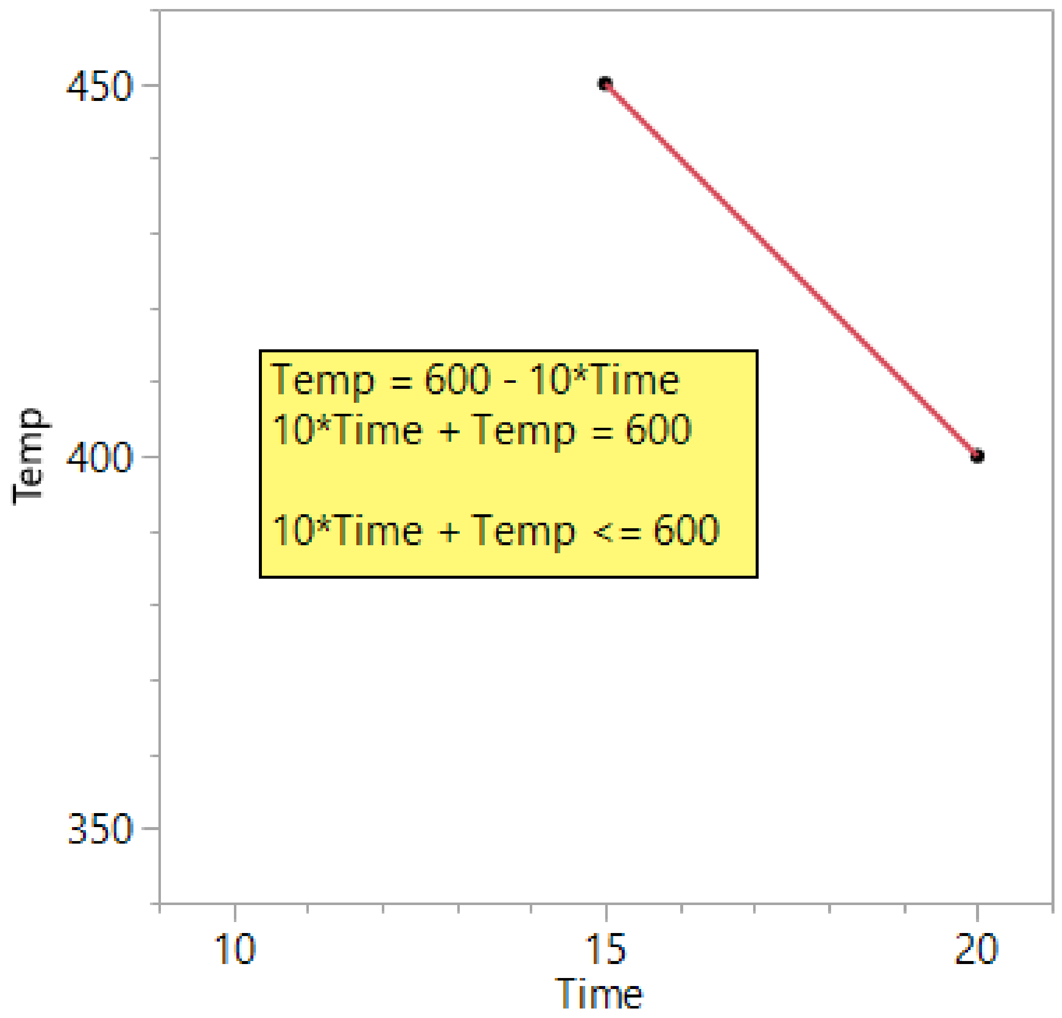
Linear Fit

Temp = 500 - 10*Time

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	500	.	.	.
Time	-10	.	.	.

Bivariate Fit of Temp By Time Constraint Location=Upper



Linear Fit

Linear Fit

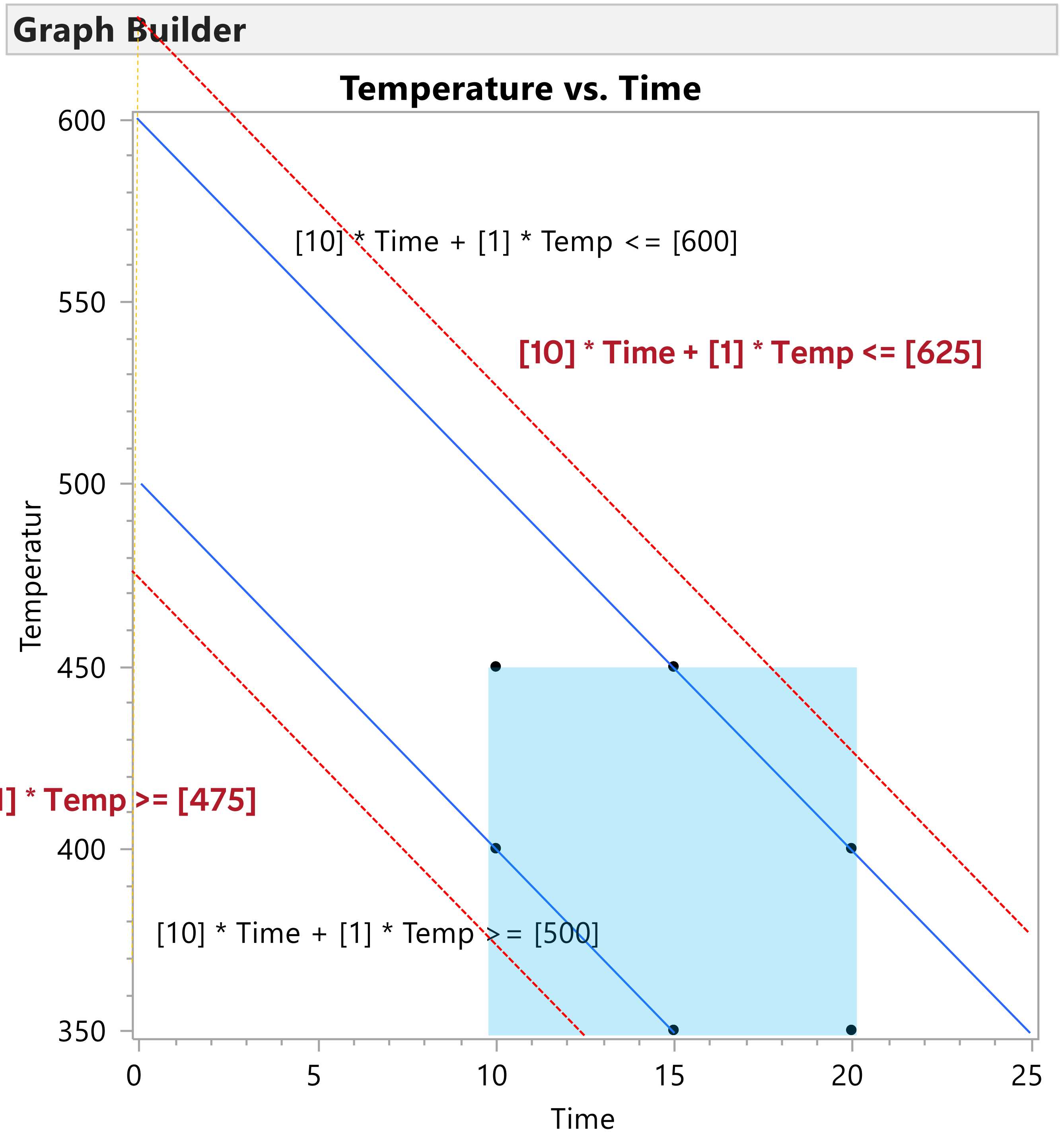
Temp = 600 - 10*Time

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	600	.	.	.
Time	-10	.	.	.

Time and Temperature Constraints Uncoded "Broadened" Design

What if constraints narrowed design region to a thin diagonal slice in Time & Temp? They would then be highly correlated.



[10] * Time + [1] * Temp >= [475]

Slope = m = rise/run = -150/15; m = -10

Intercept = b = *y* when *x* = zero; b = 625

Intercept = b = *y* when *x* = zero; b = 475

y = mx + b

Temp = m*Time + b

[1]*Temp = [-10]*Time + [625]

[10]*Time + [1]*Temp = [625]

[10]*Time + [1]*Temp <= [625]

y = mx + b

Temp = m*Time + b

[1]*Temp = [-10]*Time + [475]

[10]*Time + [1]*Temp = [475]

[10]*Time + [1]*Temp >= [475]

4 Factors, 3 Types, 1 Hard-to-Change, Plus 2 constraints

Create DOE for this “Real-World” Pizza Process

Factors

Add Factor | Remove | Add N Factors 1

Name	Role	Changes	Values			
Time	Continuous	Easy	10		20	
Temperature	Continuous	Hard	350		450	
Pizza Size	Discrete Num	Easy	9	12	14	16
Pizza Type	Categorical	Easy	Cheese	Veggies	Meats	

Define Factor Constraints

- None
- Specify Linear Constraints
- Use Disallowed Combinations Filter
- Use Disallowed Combinations Script

Linear Constraints

Add

10 Time + 1 Temperature ≤ 625

10 Time + 1 Temperature ≥ 475

Remove Last Constraint

Check Constraints

Model

Main Effects | Interactions | RSM | Cross | Powers | Remove Term

Name	Estimability
Intercept	Necessary
Time	Necessary
Temperature	Necessary
Pizza Size	Necessary
Pizza Size*Pizza Size	If Possible
Pizza Size*Pizza Size*Pizza Size	If Possible
Pizza Type	Necessary
Time*Time	Necessary
Time*Temperature	Necessary
Temperature*Temperature	Necessary
Time*Pizza Size	Necessary

Design Generation

Number of Whole Plots 6

Number of Runs:

- Minimum 17
- Default 24
- User Specified 24

Make Design

6/2 Cols — 24/12

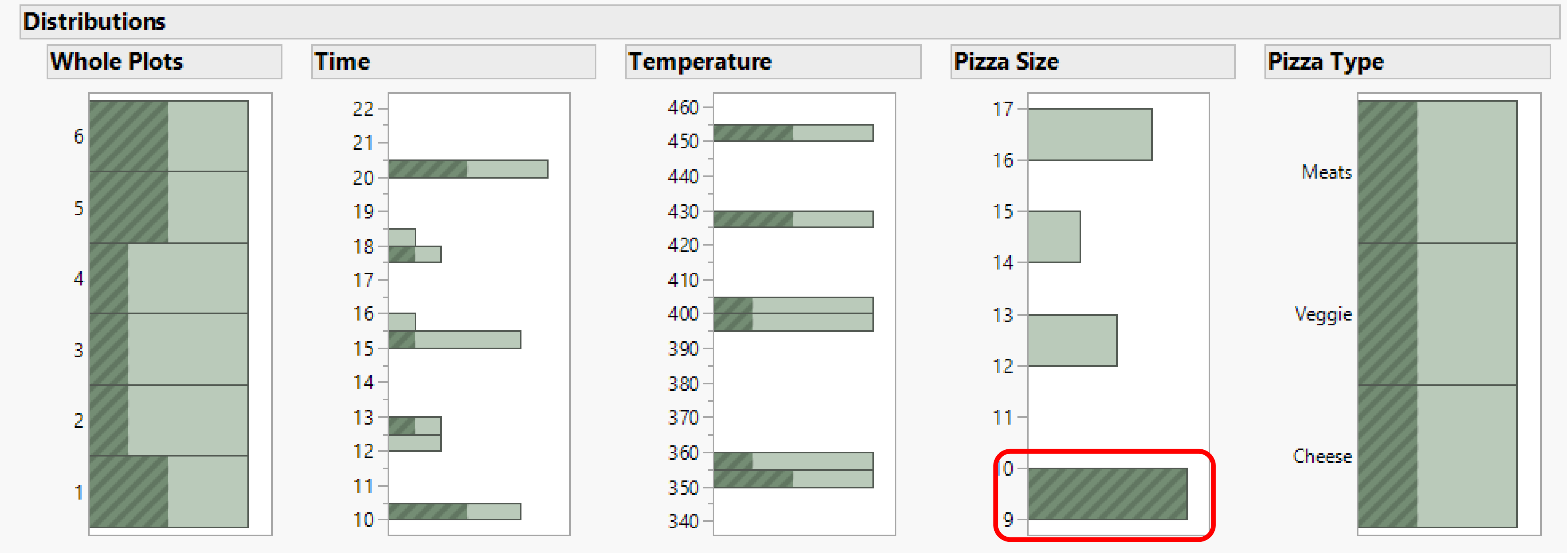
	Whole Plots	Time	Temperature	Pizza Size	Pizza Type
1	1	10	450	14	Veggie
2	1	17.5	450	9	Meats
3	1	10	450	9	Cheese
4	1	17.5	450	16	Cheese
5	2	15	397.5	14	Cheese
6	2	10	397.5	16	Cheese
7	2	10	397.5	9	Veggie
8	2	15	397.5	12	Meats
9	3	12	355	16	Veggie
10	3	20	355	16	Cheese
11	3	15.2	355	12	Meats
12	3	20	355	9	Veggie
13	4	20	400	14	Meats
14	4	18	400	16	Veggie
15	4	15	400	12	Cheese
16	4	10	400	9	Meats
17	5	15	425	9	Veggie
18	5	12	425	16	Meats
19	5	20	425	9	Cheese
20	5	20	425	12	Veggie
21	6	12.5	350	16	Meats
22	6	12.5	350	9	Cheese
23	6	20	350	9	Meats
24	6	15.5	350	12	Veggie

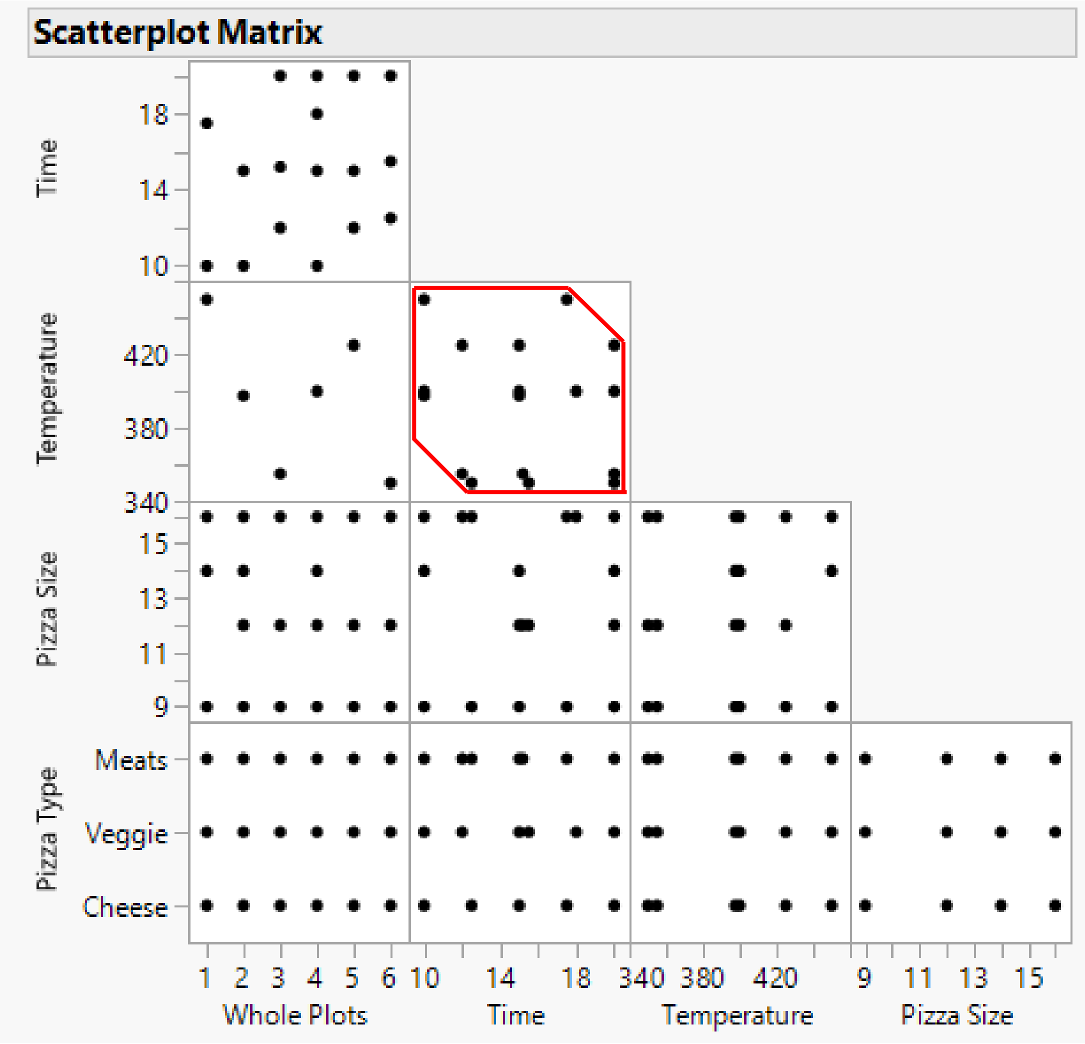
Visualize Design Balance

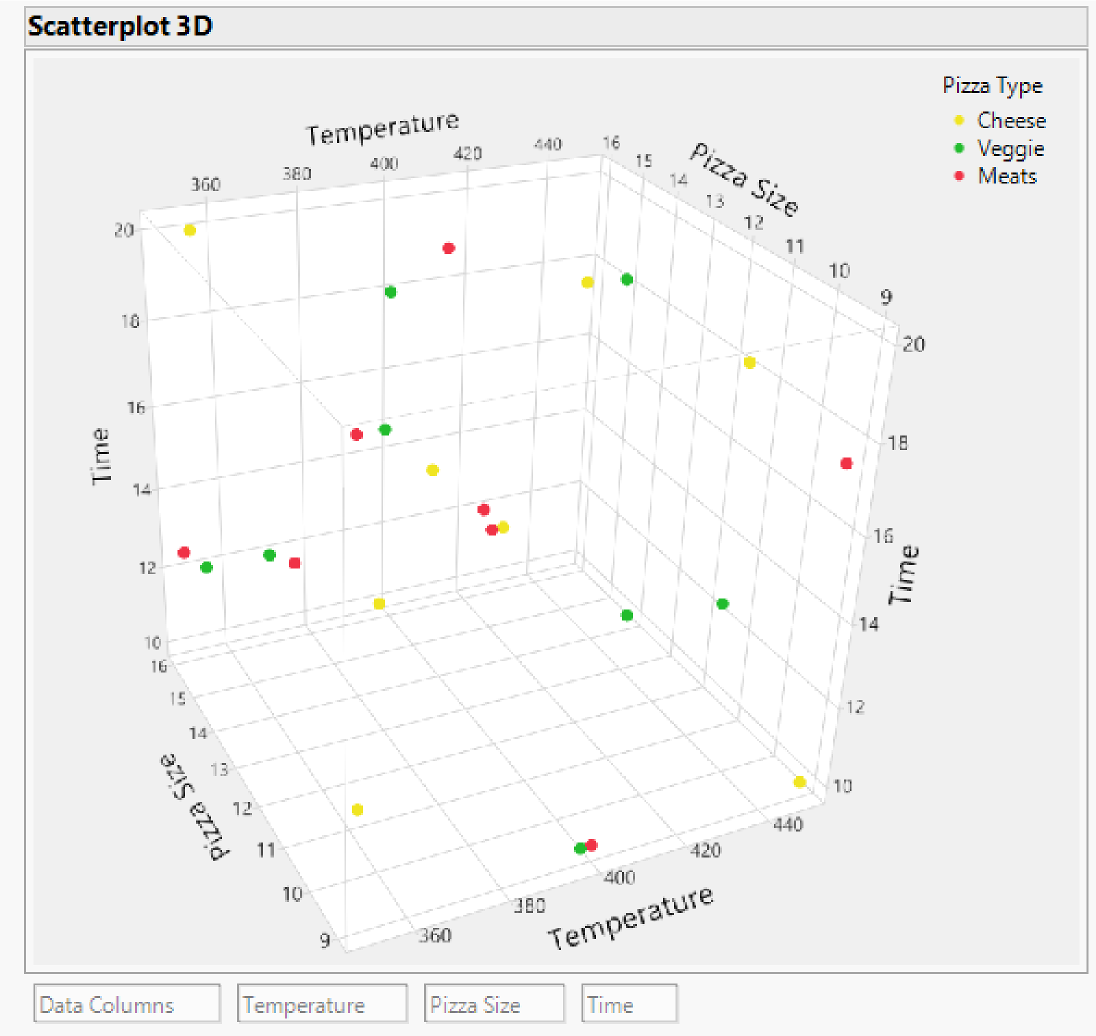
Distribution of Design Trials & Projections of Designs Trials in 2-D & 3-D

6/2 Cols
24/12

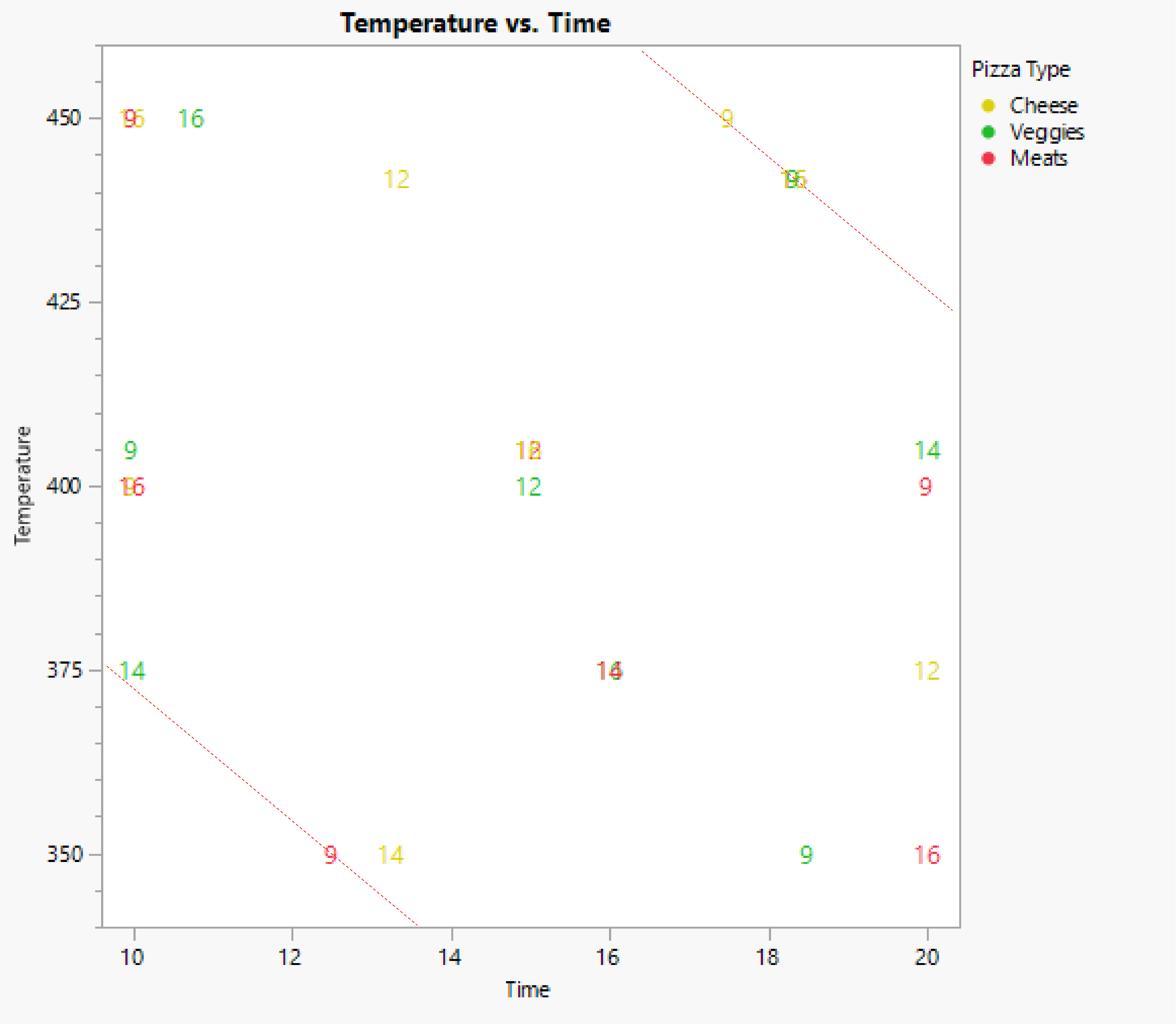
	Whole Plots	Time	Temperature	Pizza Size	Pizza Type
1	1	10	450	14	Veggie
2	1	17.5	450	9	Meats
3	1	10	450	9	Cheese
4	1	17.5	450	16	Cheese
5	2	15	397.5	14	Cheese
6	2	10	397.5	16	Cheese
7	2	10	397.5	9	Veggie
8	2	15	397.5	12	Meats
9	3	12	355	16	Veggie
10	3	20	355	16	Cheese
11	3	15.2	355	12	Meats
12	3	20	355	9	Veggie
13	4	20	400	14	Meats
14	4	18	400	16	Veggie
15	4	15	400	12	Cheese
16	4	10	400	9	Meats
17	5	15	425	9	Veggie
18	5	12	425	16	Meats
19	5	20	425	9	Cheese
20	5	20	425	12	Veggie
21	6	12.5	350	16	Meats
22	6	12.5	350	9	Cheese
23	6	20	350	9	Meats
24	6	15.5	350	12	Veggie



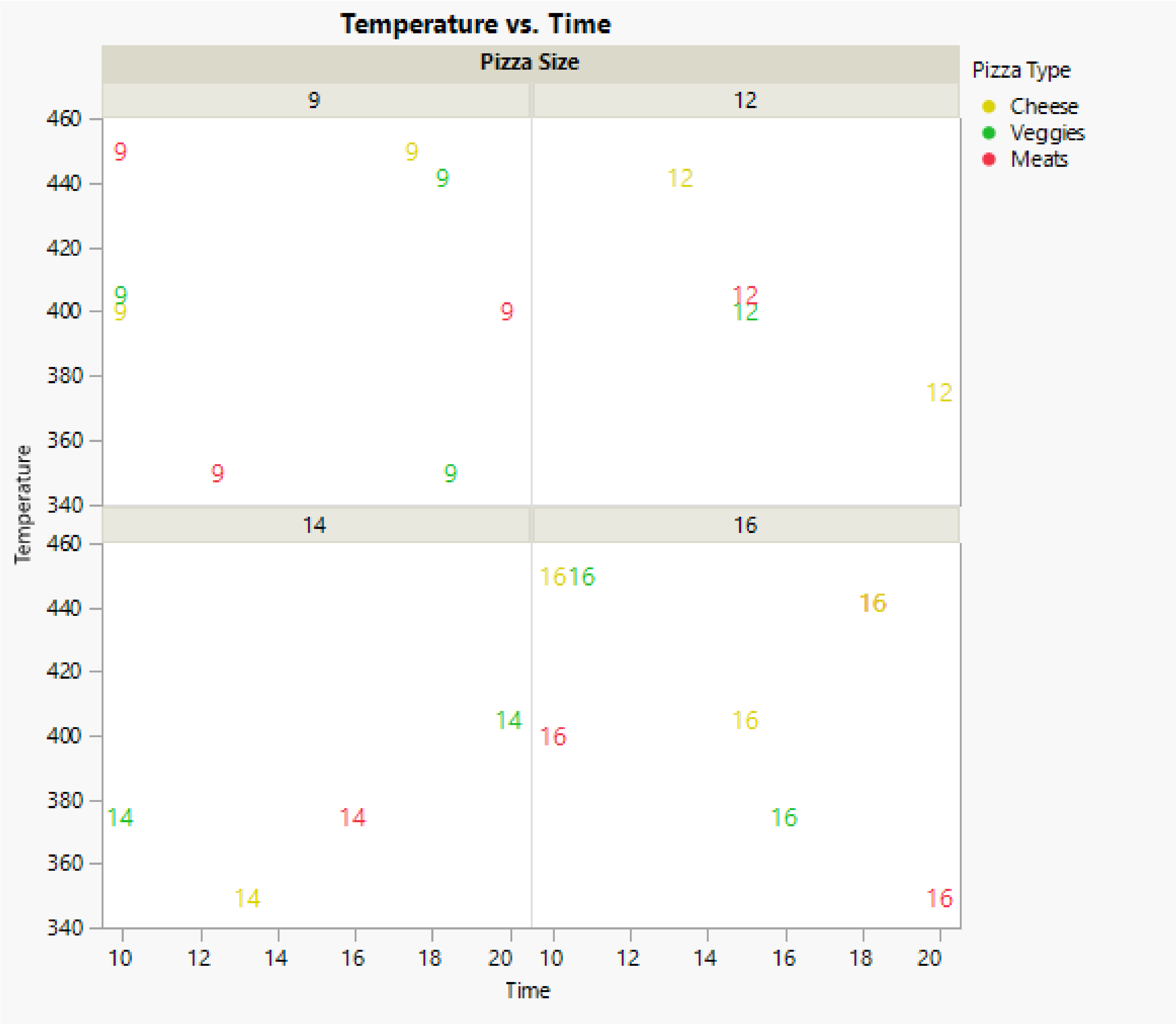




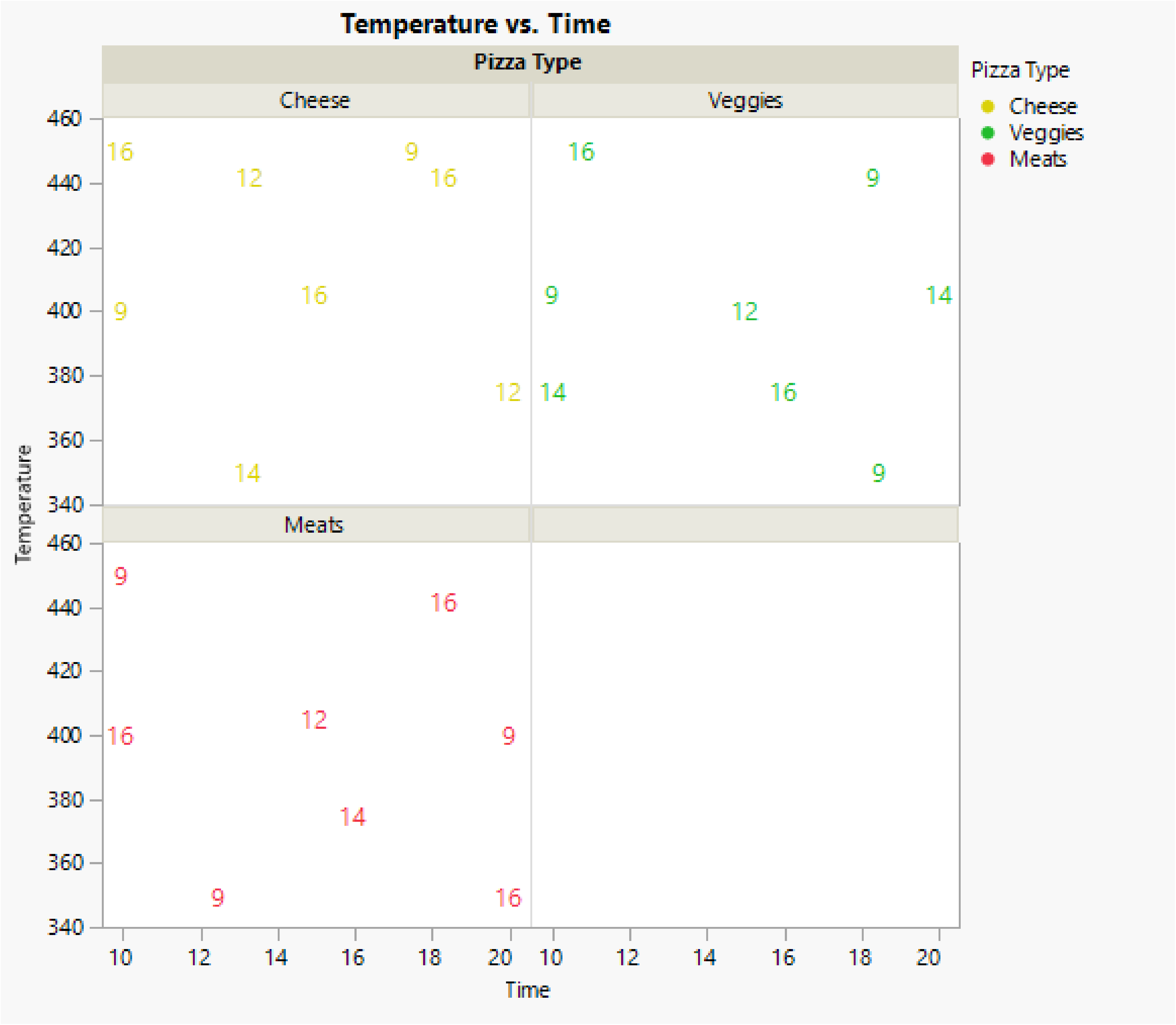
Final Design Showing Constrained Regions



Final Design Showing Constrained Regions



Final Design Showing Constrained Regions



Agenda

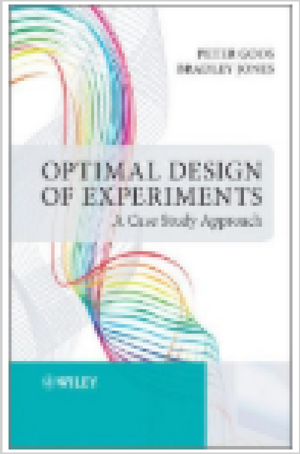
- Multiple Response Optimization

  Trade-Space Analysis – Why we do Design of Experiments (DOE)
- Six step framework for creating a successful DOE
  & important questions to consider
- Real-World Experimental Issues – Custom DOE is all about

  Making Designs Fit the Problem –
  NOT Making Problems Fit the Designs!
- Two Example Designs – 1st Quick (slide), 2nd Detailed (run JMP)
  1. Four continuous factors, three responses, and 2nd order RSM model
  2. Continuous, discrete numeric, categorical, and hard-to-change factors, plus added constraints, and 2nd order RSM model

DOE BOOKS

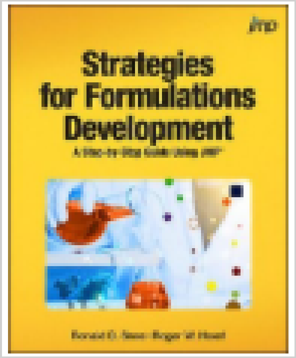
www.jmp.com/books



› Optimal Design of Experiments: A Case Study Approach

by Peter Goos and Bradley Jones

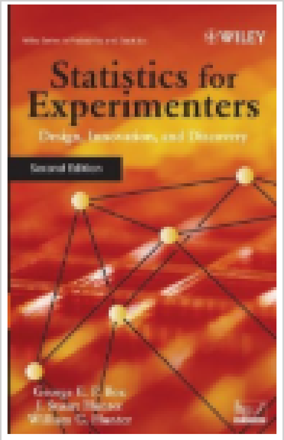
2011 (John Wiley Sons Inc.)



› Strategies for Formulations Development: A Step-by-Step Guide Using JMP

by Ronald Snee and Roger Hoerl

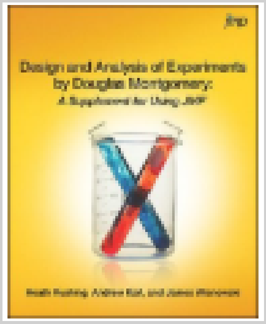
2016 (SAS Institute)



› Statistics for Experimenters: Design, Innovation, and Discovery, 2nd Edition

by George E. P. Box, J. Stuart Hunter, and William G. Hunter

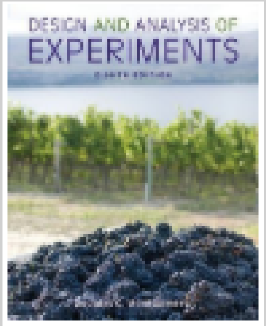
2005 (Wiley)



› Design and Analysis of Experiments by Douglas Montgomery: A Supplement for Using JMP

by Heath Rushing, James Wisnowski, and Andrew Karl

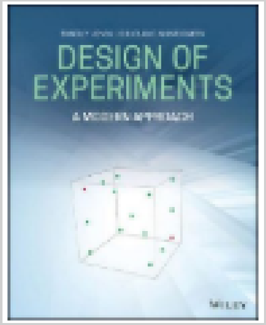
2013 (SAS Institute)



› Design and Analysis of Experiments, 8th Edition

by Douglas C. Montgomery

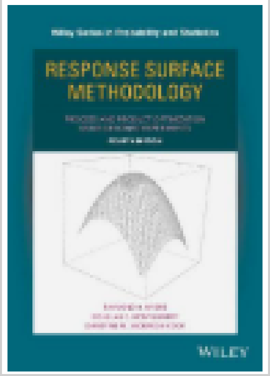
2012 (Wiley)



› Design of Experiments: A Modern Approach

by Bradley Jones and Douglas C. Montgomery

2019 (SAS Institute)



› Response Surface Methodology: Process and Product Optimization Using Designed Experiments, 4th Edition

by Raymond H. Myers, Douglas C. Montgomery, and Christine M. Anderson-Cook

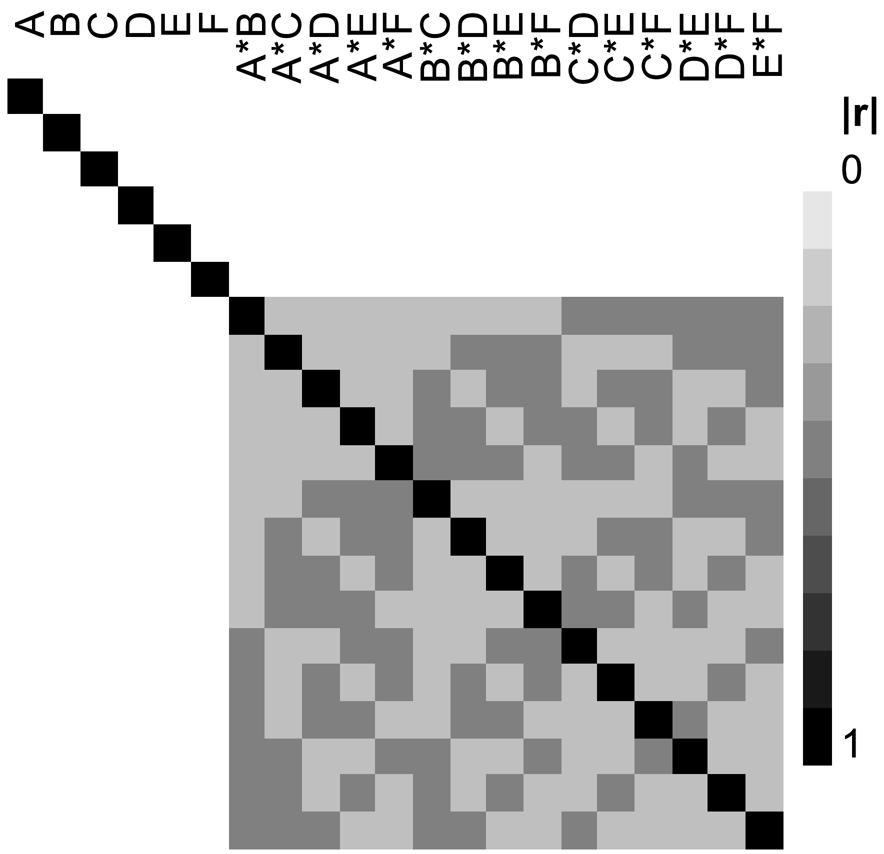
2016 (Wiley)

Definitive Screening Design (DSD)

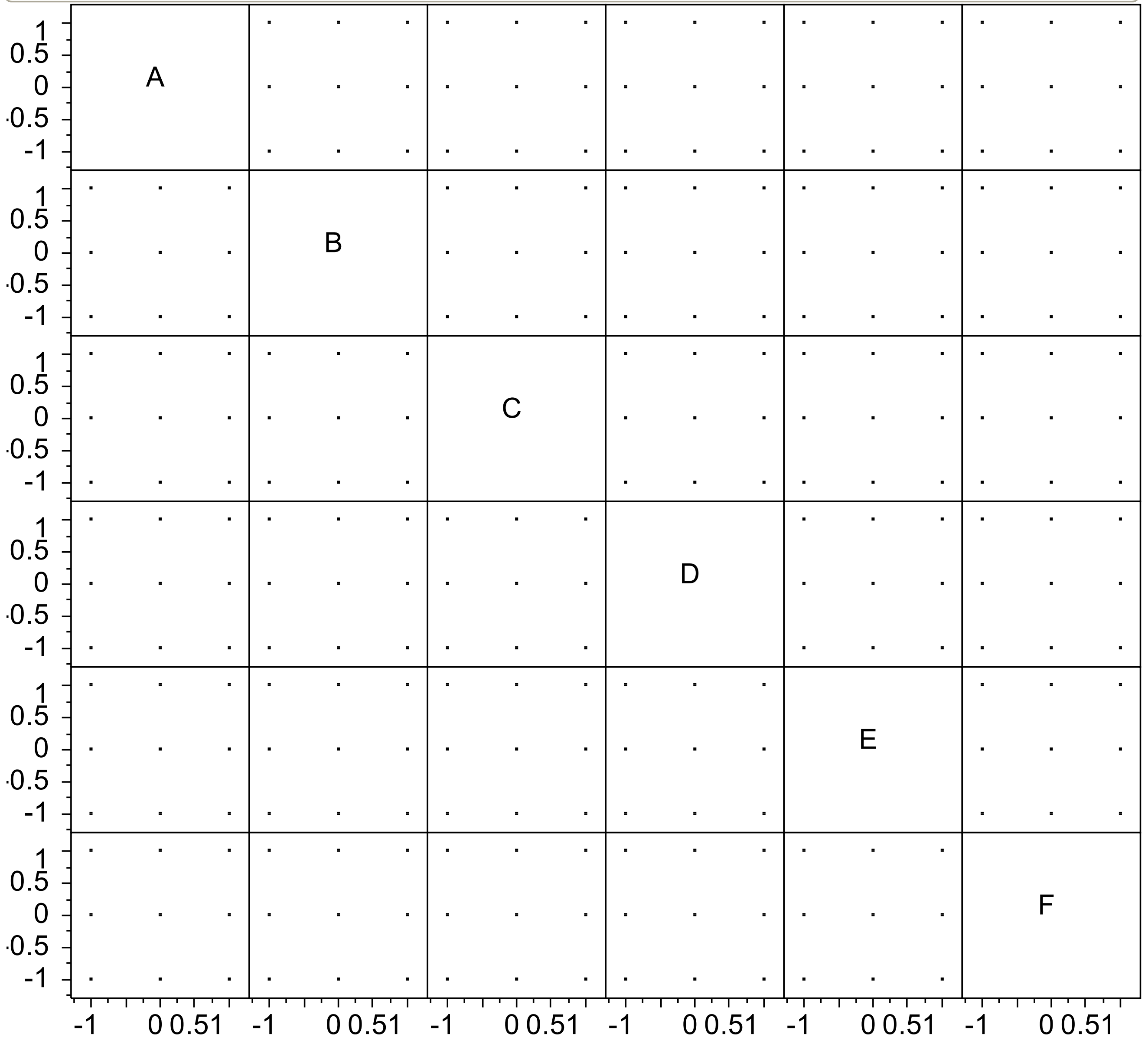
Attractive alternative to Classic 2-level screening designs

	A	B	C	D	E	F
1	0	1	-1	-1	-1	-1
2	0	-1	1	1	1	1
3	1	0	-1	1	1	-1
4	-1	0	1	-1	-1	1
5	-1	-1	0	1	-1	-1
6	1	1	0	-1	1	1
7	-1	1	1	0	1	-1
8	1	-1	-1	0	-1	1
9	1	-1	1	-1	0	-1
10	-1	1	-1	1	0	1
11	1	1	1	1	-1	0
12	-1	-1	-1	-1	1	0
13	0	0	0	0	0	0

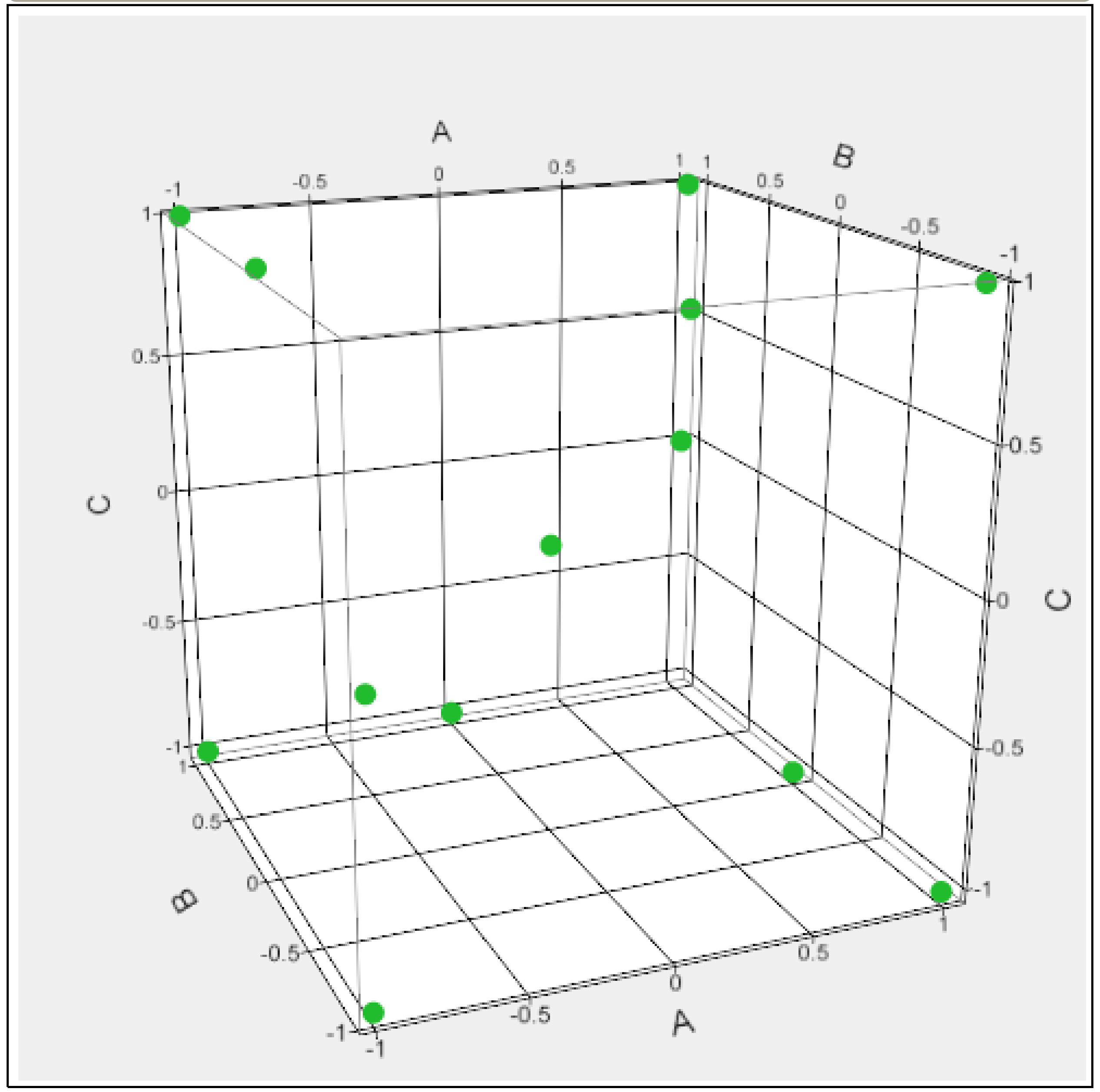
Color Map On Correlations



Scatterplot Matrix



Scatterplot 3D



Data Columns A B C

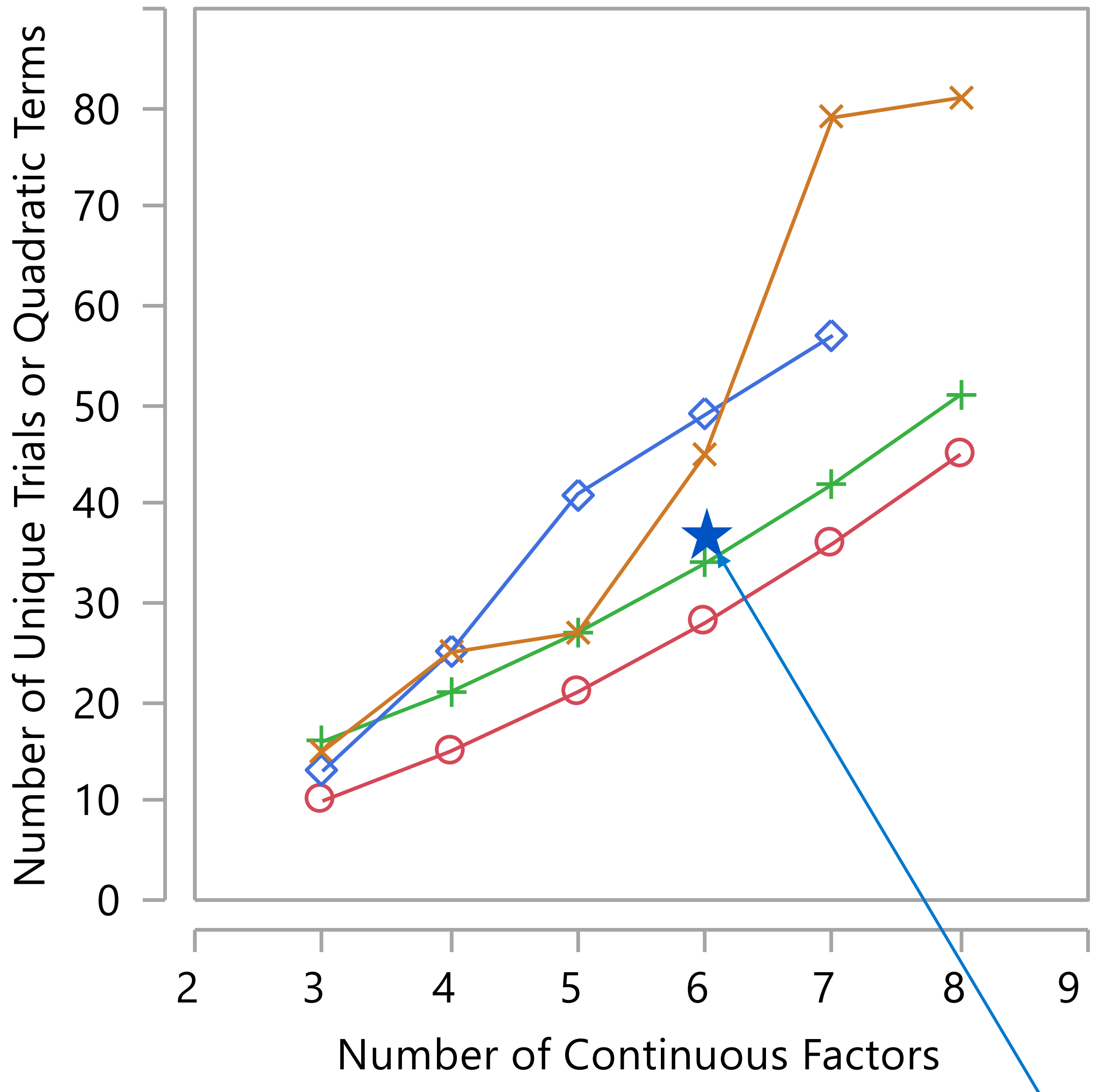
Augmentation via Custom DOE

If more than a few factors are significant for DSD, then augment design to support 2nd order model

	A	B	C	D	F	G	Block	Yield @ Time t
14	0	0	0	0	0	0	1	7.49
15	1	1	-1	1	-1	1	1	0.98
16	1	1	1	-1	-1	0	1	0.86
17	-1	1	-1	-1	1	1	1	1.25
18	1	-1	1	1	-1	-1	1	1.03
19	1	1	0	-1	1	-1	1	1.07
20	0	0	0	0	0	0	1	7.33
21	1	-1	-1	0	1	-1	1	2.61
22	-1	-1	0	1	-1	1	1	11.39
23	-1	0	1	-1	1	1	1	12.96
24	1	1	-1	1	1	1	1	1.18
25	1	0	1	1	-1	1	2	•
26	1	-1	0	1	1	0	2	•
27	1	-1	-1	1	0	1	2	•
28	1	-1	0	-1	0	-1	2	•
29	1	0	-1	-1	1	0	2	•
30	1	1	0	-1	0	1	2	•
31	1	0	1	0	1	-1	2	•
32	-1	-1	0	0	1	1	2	•
33	0	0	1	1	-1	-1	2	•
34	-1	-1	1	0	0	0	2	•
35	0	1	1	0	1	0	2	•
36	0	1	-1	1	1	-1	2	•

NOTE: First 13 rows of original design are not shown.

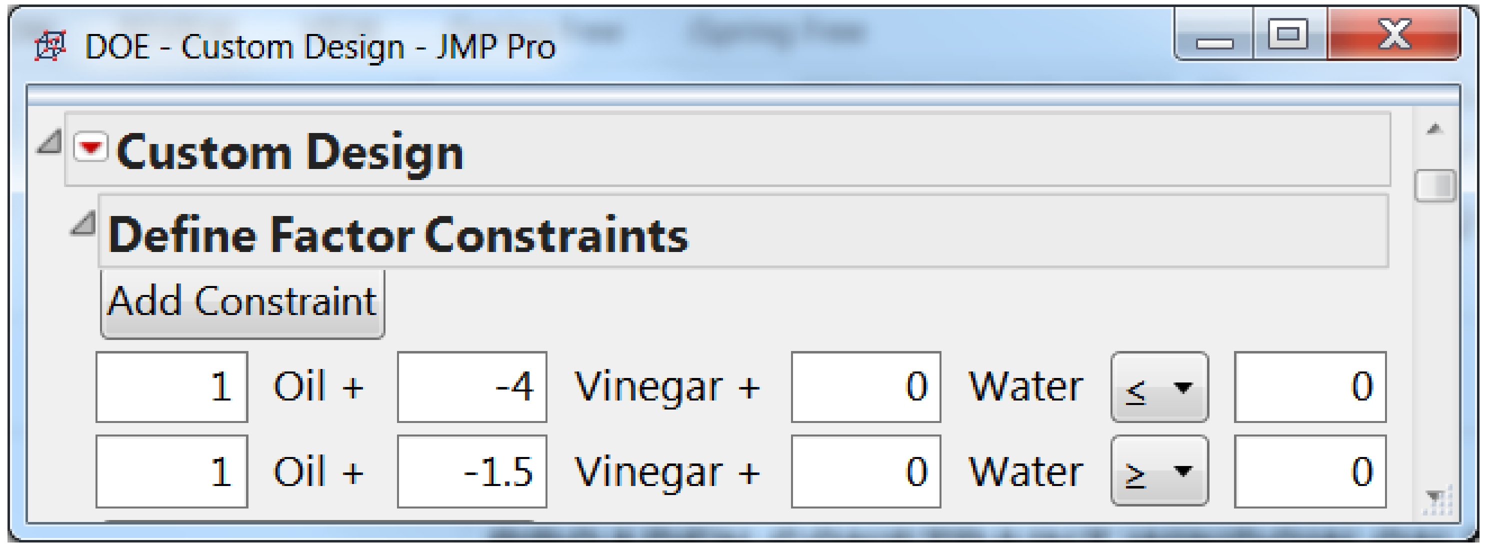
These 12 trials added onto original 24 trials to support full quadratic model in 6 most important factors plus a block effect between original and augmented trials



36 trial I-optimal response-surface design started as 10-factor DSD and was then augmented with 12 more trials in 6 most important factors

3-Component Mixture DOE with constraints

Rarely do components range from 0 to 1, unless taking up the slack in a blend, like water. Very often additional constraints



Mixture components in a DOE use ranges that are *proportions*:

O:	0.500 to 0.750	(½ to ¾)
W:	0.000 to 0.250	(0 to ¼)
V:	0.125 to 0.375	(⅛ to ⅜)

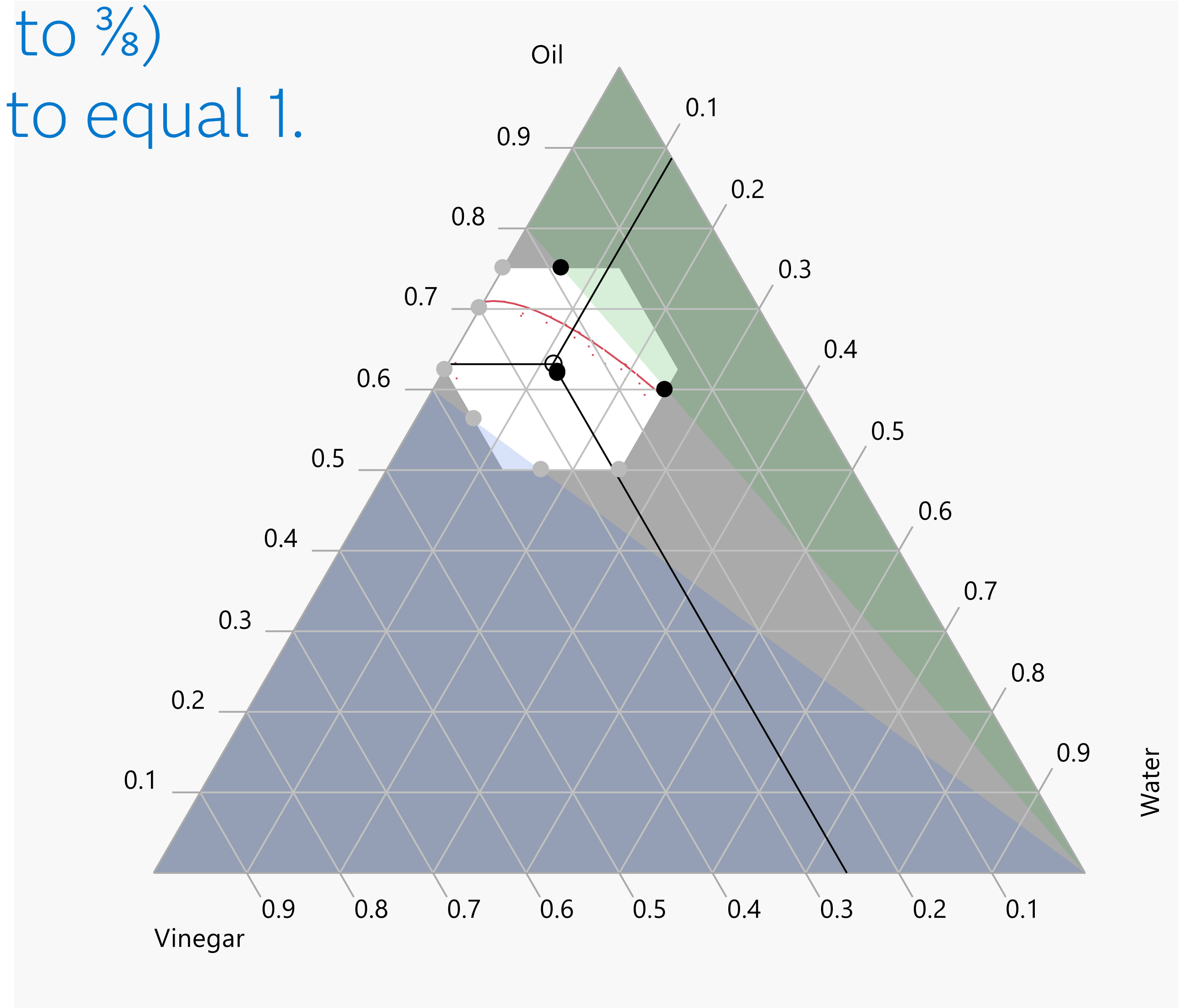
Sum of proportions *constrained* to equal 1.

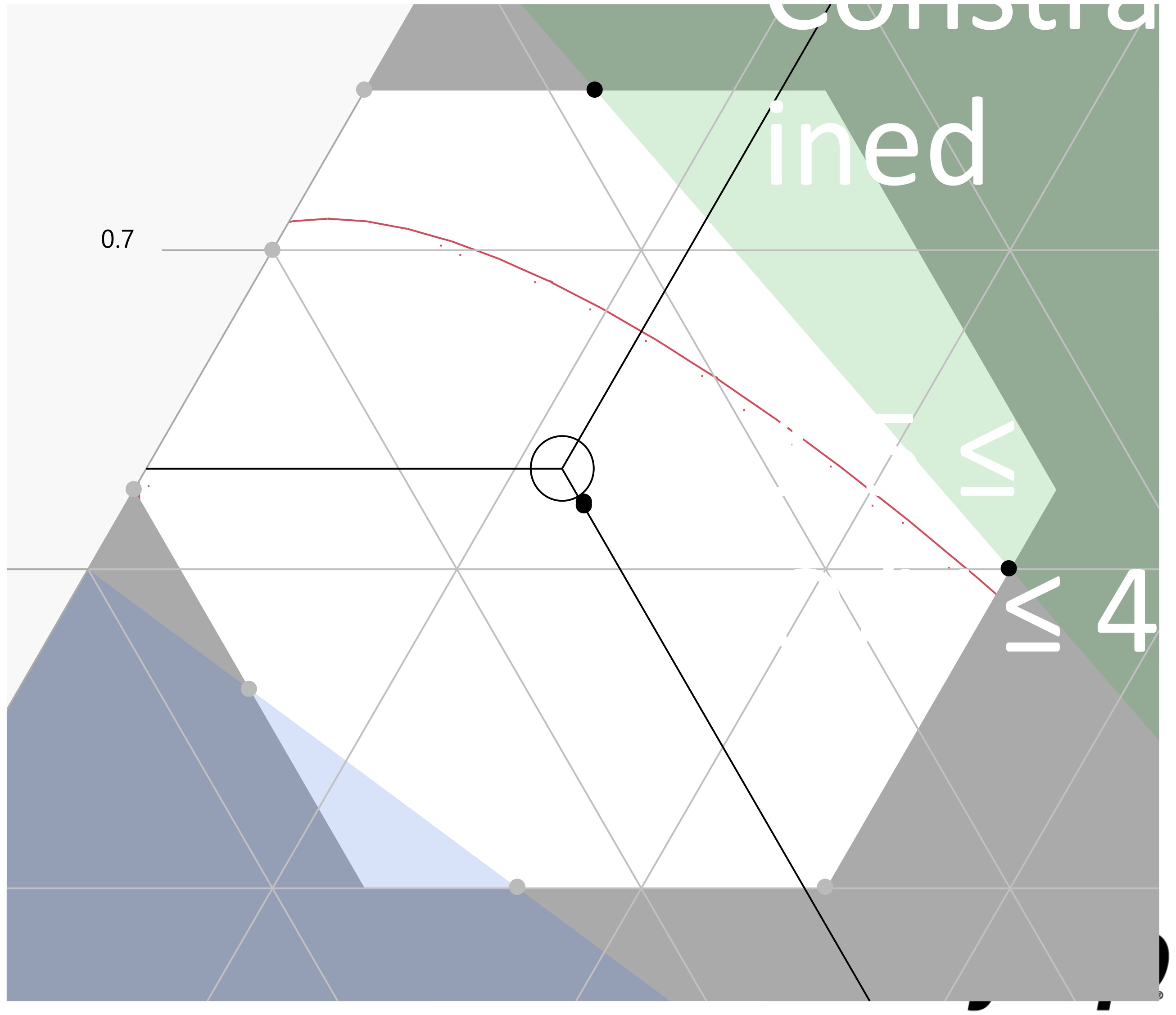
Since $1 = O + W + V$, then

$W = 1 - (O + V)$,

$O = 1 - (V + W)$, &

$V = 1 - (O + W)$

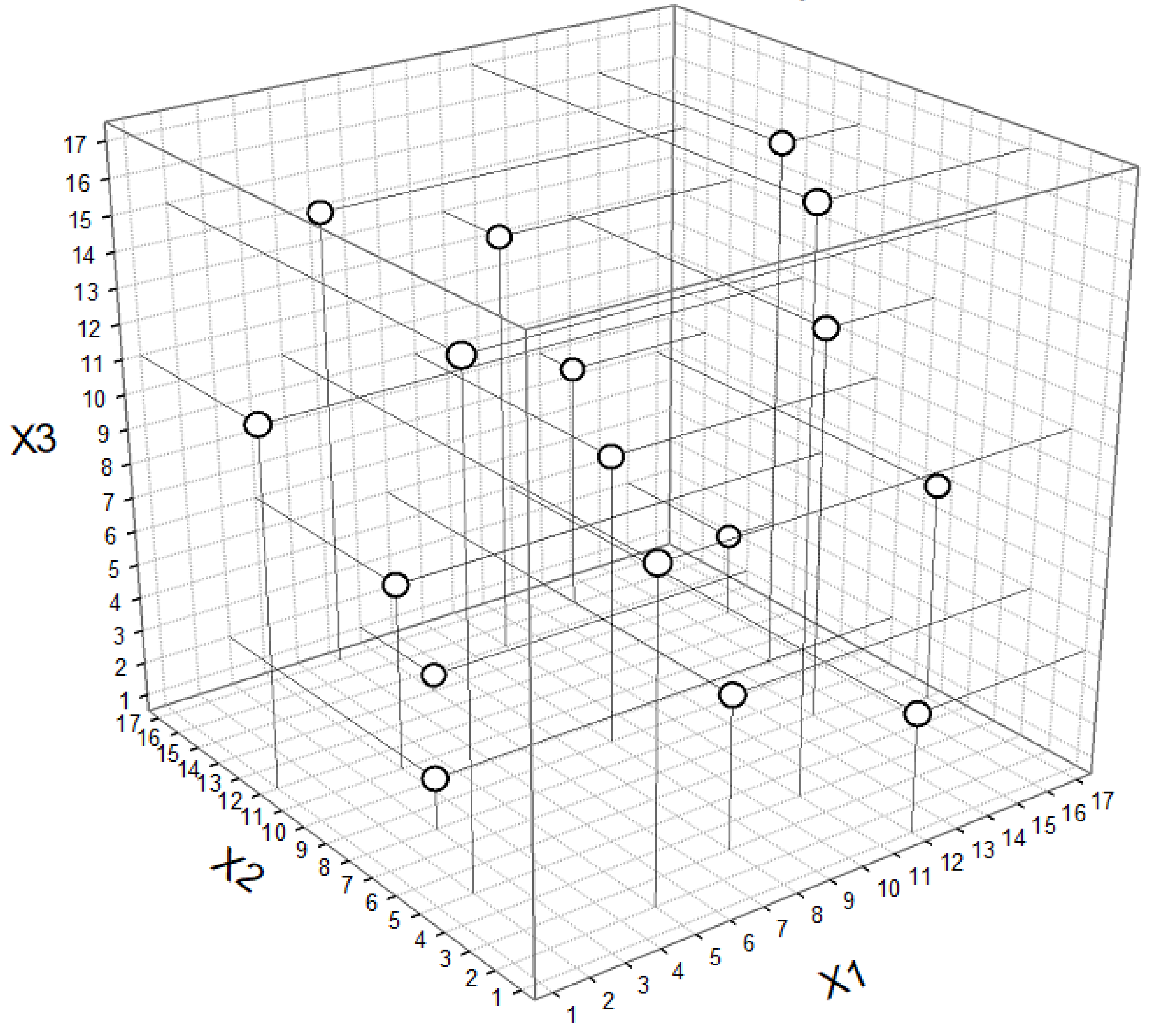




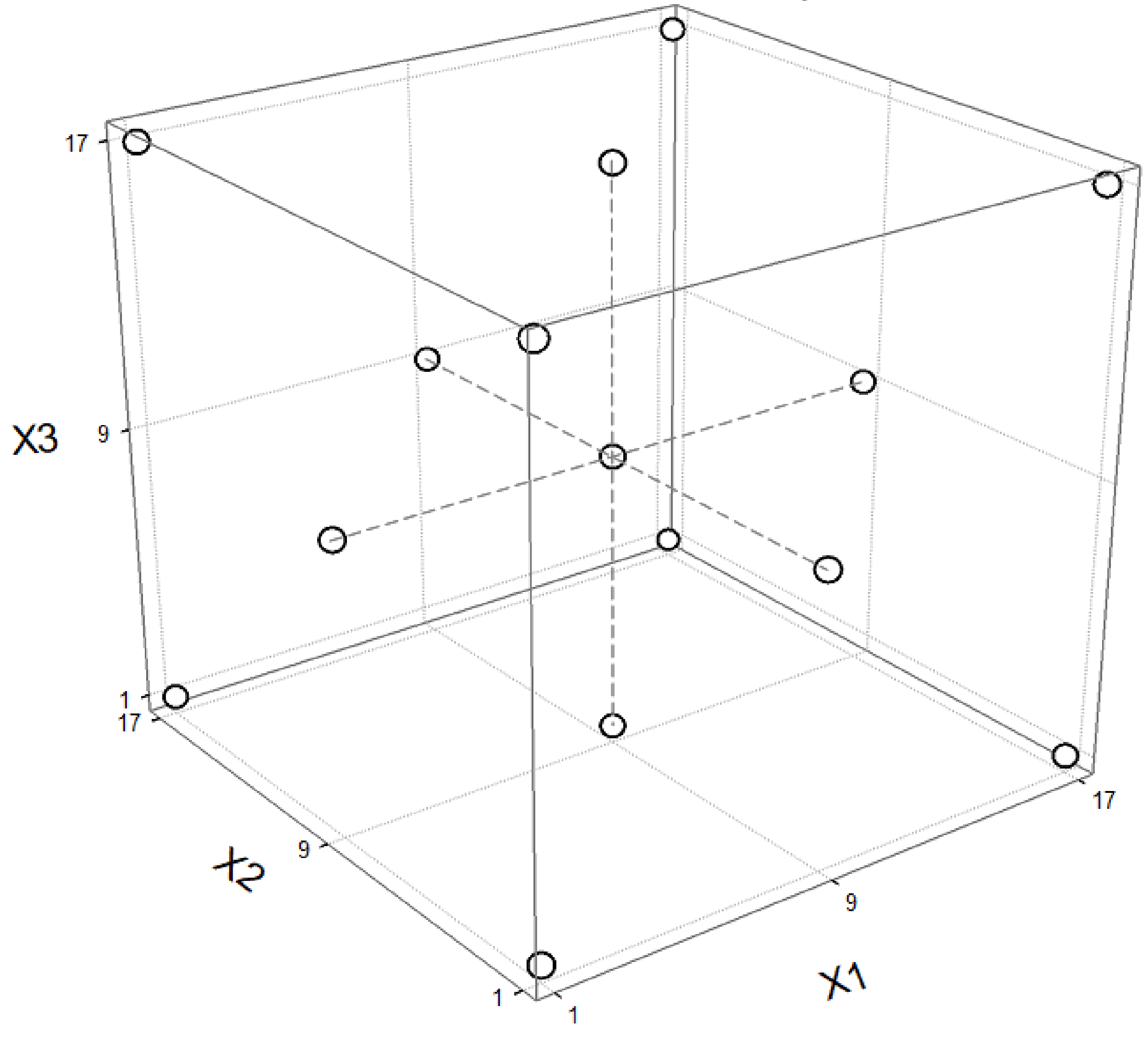
Space-filling DOE for Simulations

How are Space-Filling Designs Different from Traditional DOE?

Space-Filling Design
for 3 Variables with 17 Unique Trials



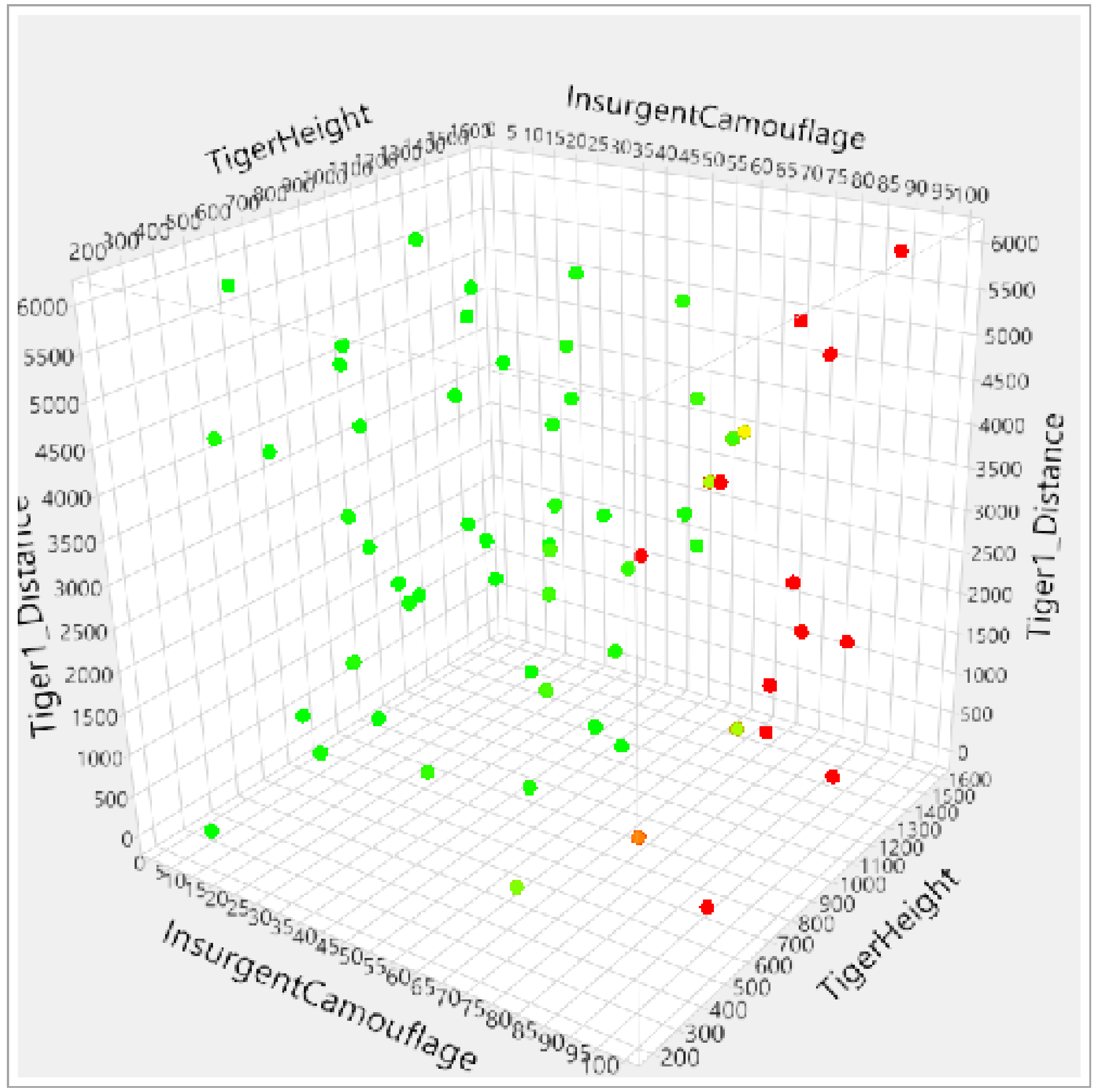
Response-Surface Design
for 3-Variables with 15 Unique Trials

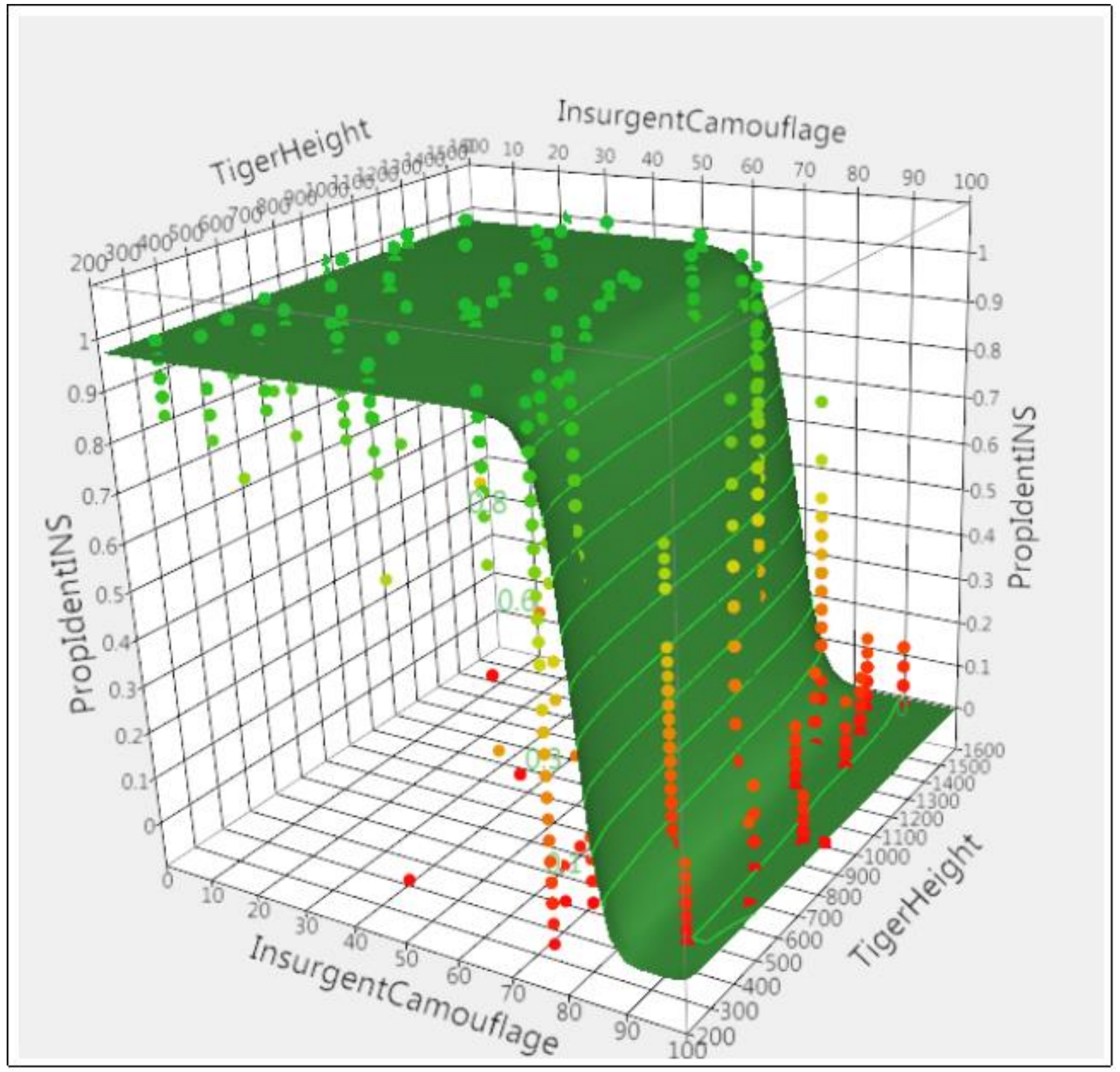


Rather than emphasizing high leverage trials ("corners") for a simple polynomial model, space-filling designs "spread" their trials more uniformly through the space to better capture the local complexities of the simulation model.

Space-filling DOE for Simulations

Space-Filling designs are better able to detect when a process falls off a cliff or has a spike

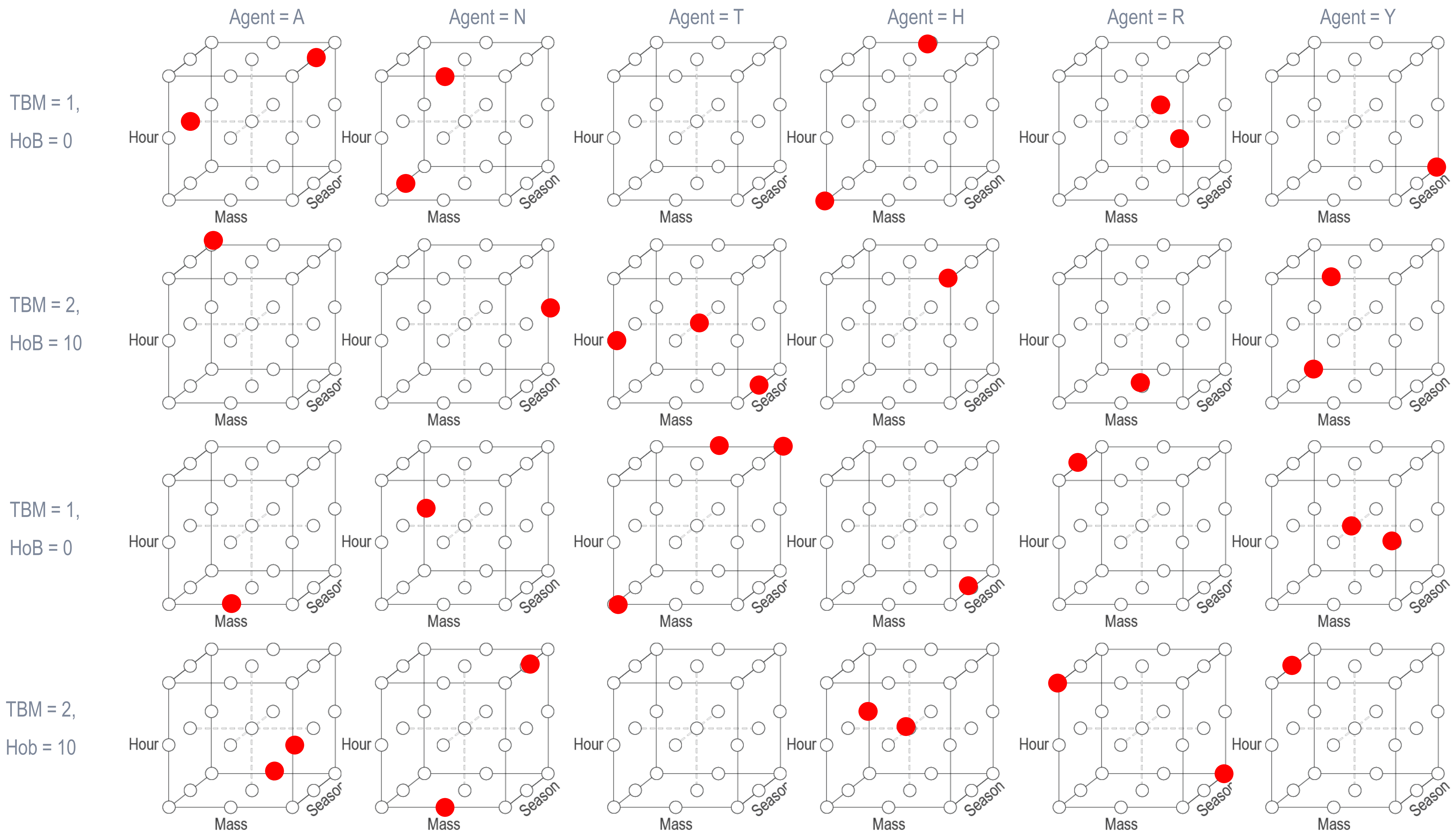




Rather than emphasizing high leverage trials ("corners") for a simple polynomial model, space-filling designs "spread" their trials more uniformly through the space to better capture the local complexities of the simulation model.

Sequential Experimentation

All 648 Possible Combinations (6 X 2 X 2 X 3 X 3 X 3) of Settings for 6 Variables with Mixed Number of Levels



Red Dots Mark the 36 Trials (an Orthogonal Array) Analyzed for Stage 1

Sequential Experimentation

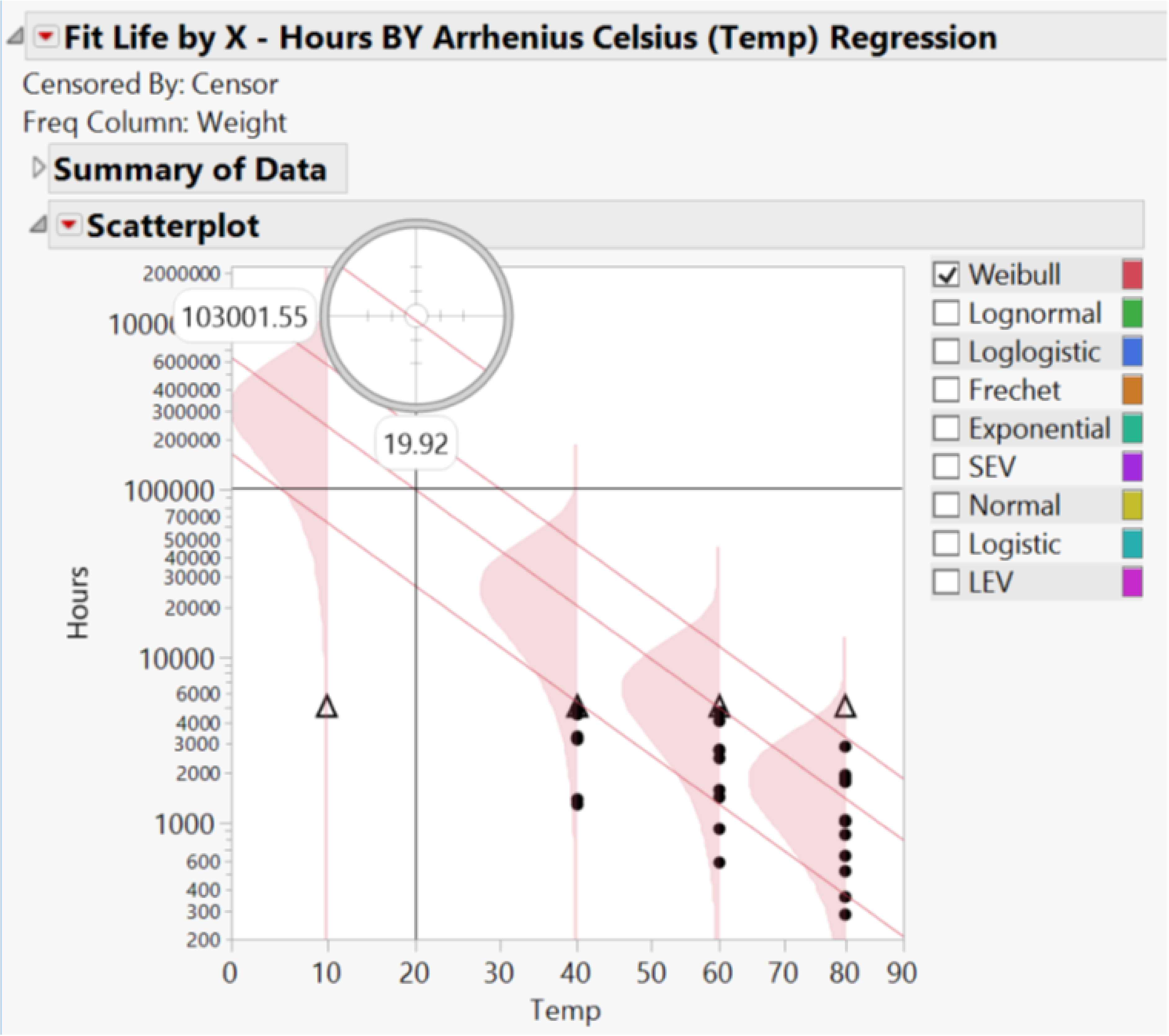
Four Stage Design supporting increasing complexity of model

Stage 1	Stage 2	Stage 3	Stage 4
36 Total Simulations	**108** Total Simulations	**324** Total Simulations	ALL **648** Simulations
Design 1, 36 trials	**Design 1, 36 trials**	**Design 1, 36 trials**	**Design 1, 36 trials**
	Design 2, 72 trials	**Design 2, 72 trials**	**Design 2, 72 trials**
		Design 3, 216 trials	**Design 3, 216 trials**
Main effects only for ALL variables + some 2-way interactions	Stage 1 effects plus all 2-way interactions + some 3-way interactions	Stage 2 effects plus all 3-way interactions	Stage 3 effects plus ALL remaining 4-way, 5-way and 6-way interactions
5.6% of 648	16.7% of 648	50% of 648	**Design 4, 324 trials** NOTE: Length of this green box should be longer than shown

324 trials in Design 4 used as checkpoints for Designs 1, 2 & 3 →

Accelerated Life Test Design

Take trials where they give the best information



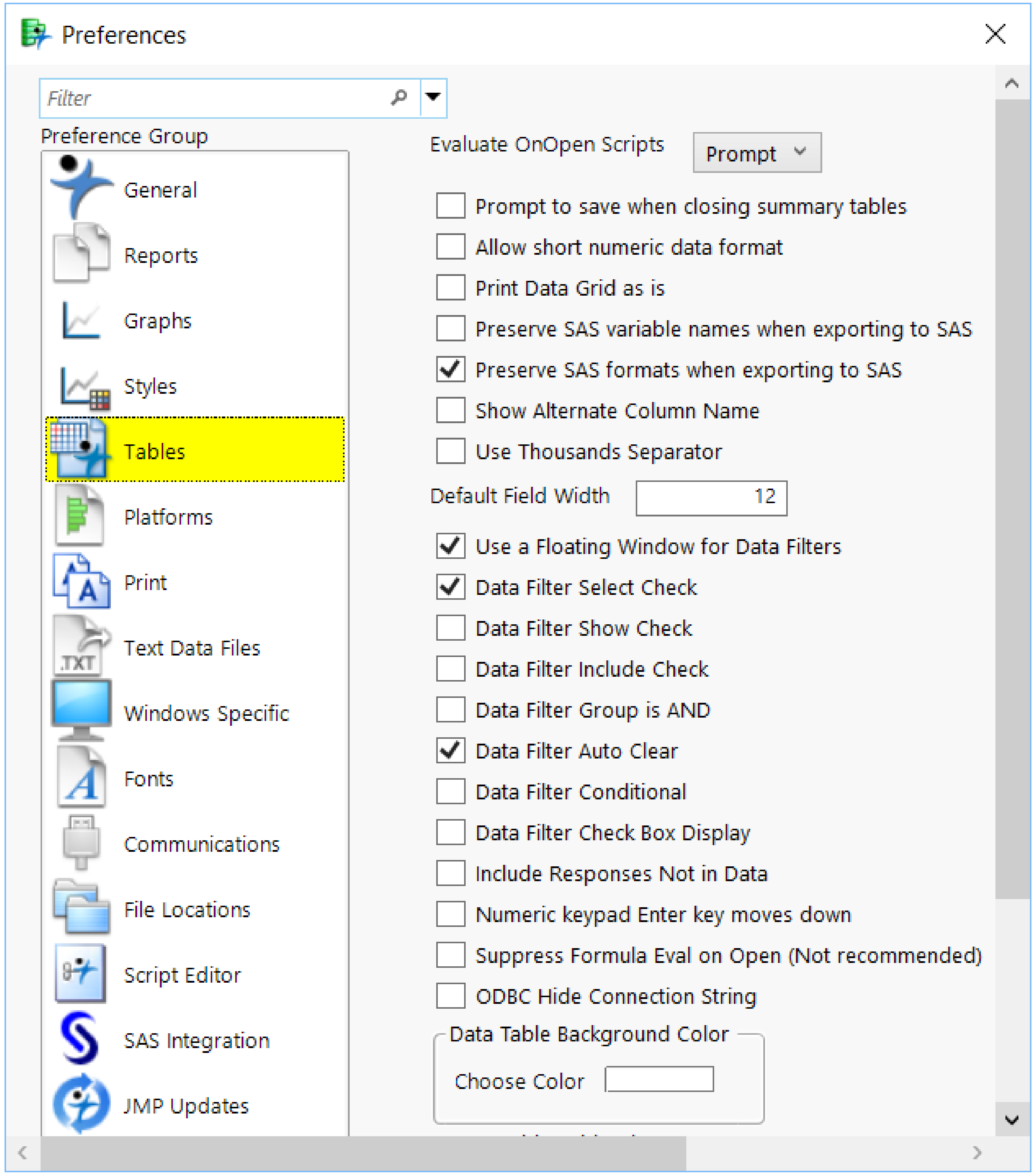
Functional Data Analysis for DOE

Modeling the “shape” of a stream of data – shape is the fundamental unit of observation – dimension reduction with functional PCA

Functional Data *Curve*

DOE Factors

Covering Arrays
Fewest Tests for N-way Coverage



Twenty check boxes in Preferences dialog

2^{20} = 1,048,576 possible combinations

How many tests required to check:

All pairs?

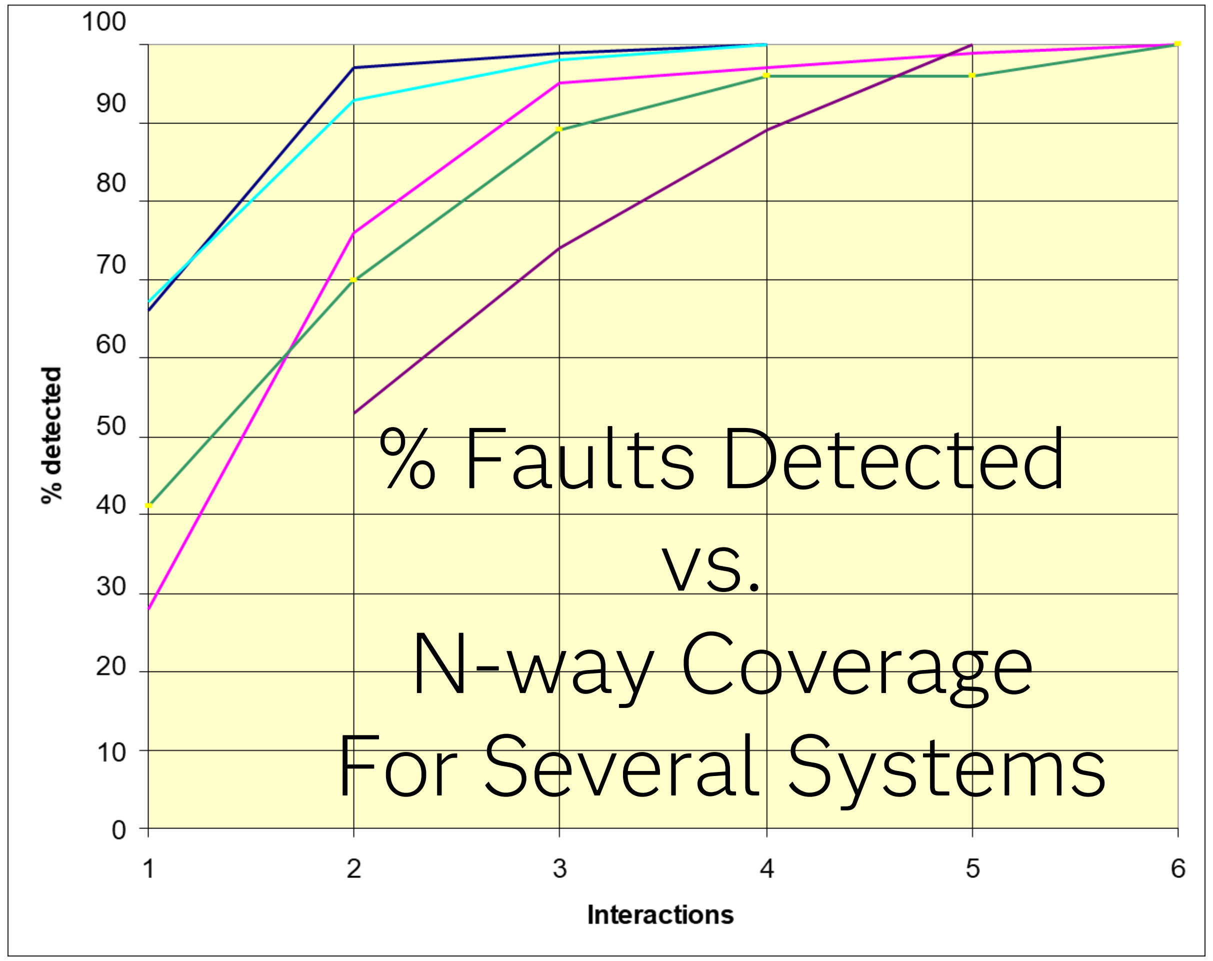
All triples?

All quadruples?

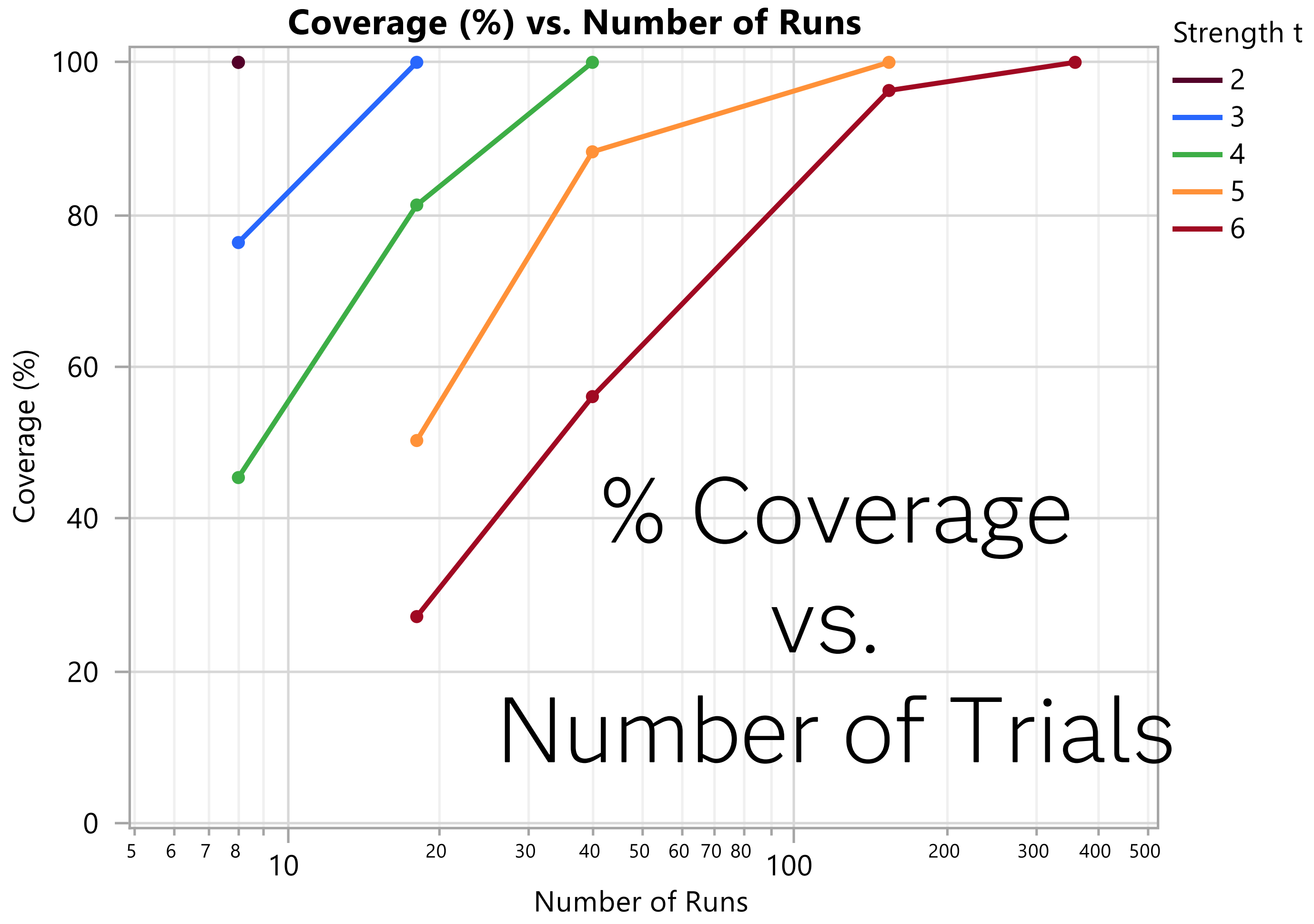
All quintuples?

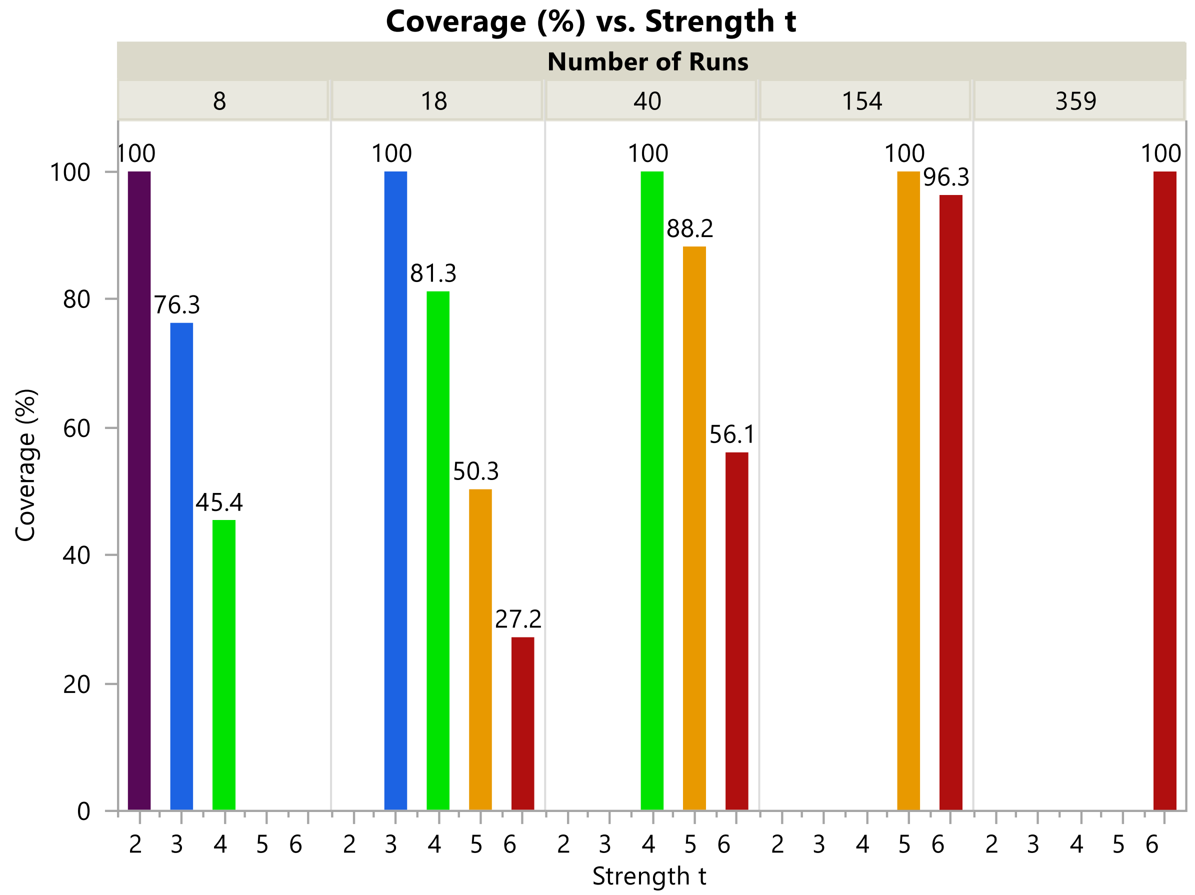
All sextuples?

Covering Arrays
Fewest Tests for
N-way Coverage



Graph courtesy of Rick Kuhn, NIST





Number of Runs: 8

t	Coverage	Diversity
2	100.00	50.00
3	76.32	76.32
4	45.43	90.87

Number of Runs: 18

t	Coverage	Diversity
3	100.00	44.44
4	81.25	72.22
5	50.30	89.42
6	27.16	96.57

Number of Runs: 40

t	Coverage	Diversity
4	100.00	40.00
5	88.24	70.59
6	56.07	89.71

Number of Runs: 154

t	Coverage	Diversity
5	100.00	20.78
6	96.39	40.06

Number of Runs: 359

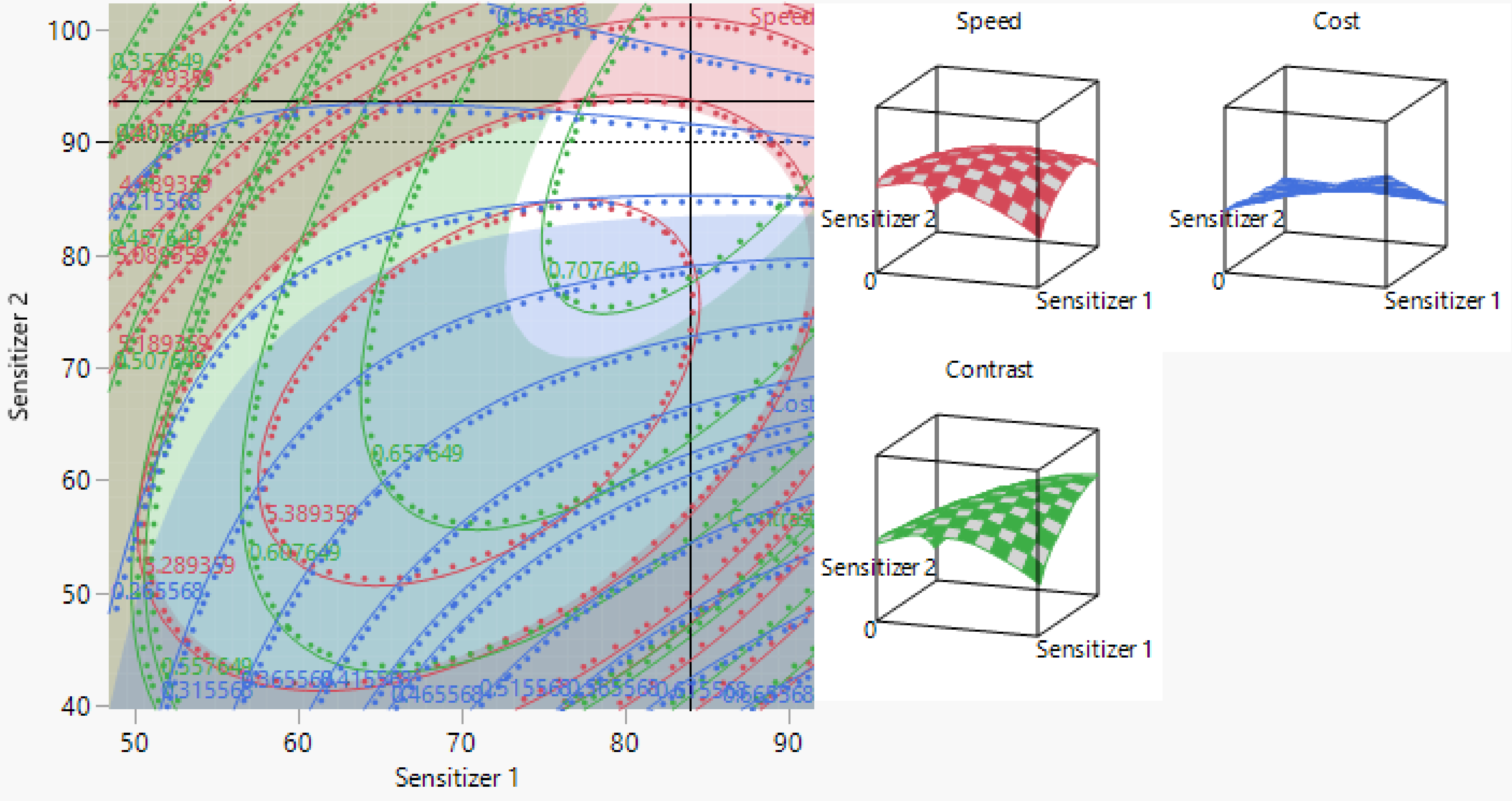
t	Coverage	Diversity
6	100.00	17.83

Trade-off & Optimization

Cost response given 6X the importance of *Speed* & *Contrast*

Factor	Current X
Sensitizer 1	84.016038
Sensitizer 2	93.725925
Dye	283.02514
Reaction Time	120

Response	Contour	Current Y	Lo Limit	Hi Limit
Speed	5.15	5.2902084	5.3	.
Contrast	0.55	0.72549	0.7	.
Cost	0.45	0.1991539	.	0.28

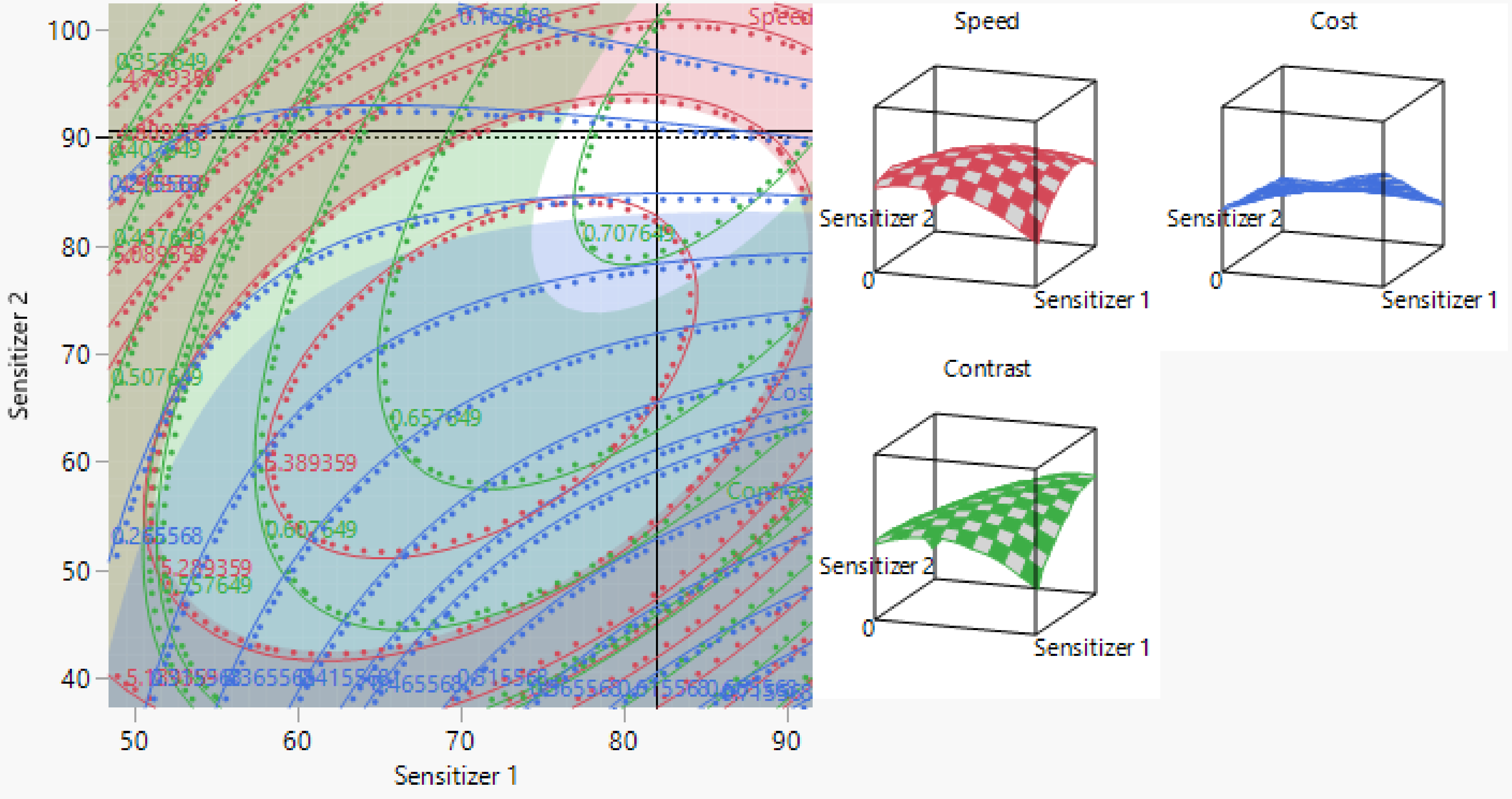


Trade-off & Optimization

After selecting location in the "Acceptable White Region" of the overlaid contour plots

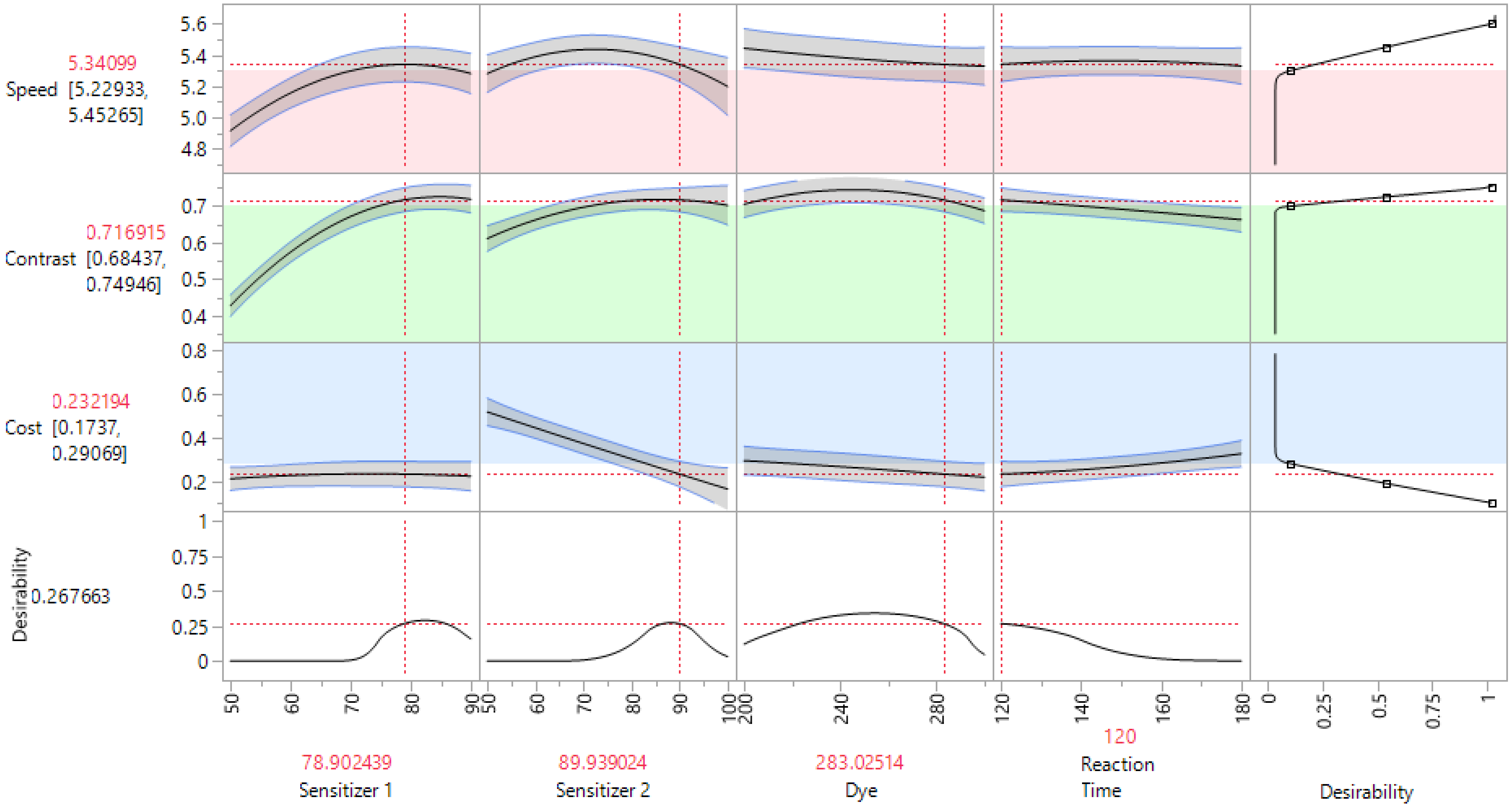
Factor	Current X
Sensitizer 1	81.958309
Sensitizer 2	90.706277
Dye	286.82246
Reaction Time	120

Response	Contour	Current Y	Lo Limit	Hi Limit
Speed	5.15	5.3269582	5.3	.
Contrast	0.55	0.7177857	0.7	.
Cost	0.45	0.2211116	.	0.28

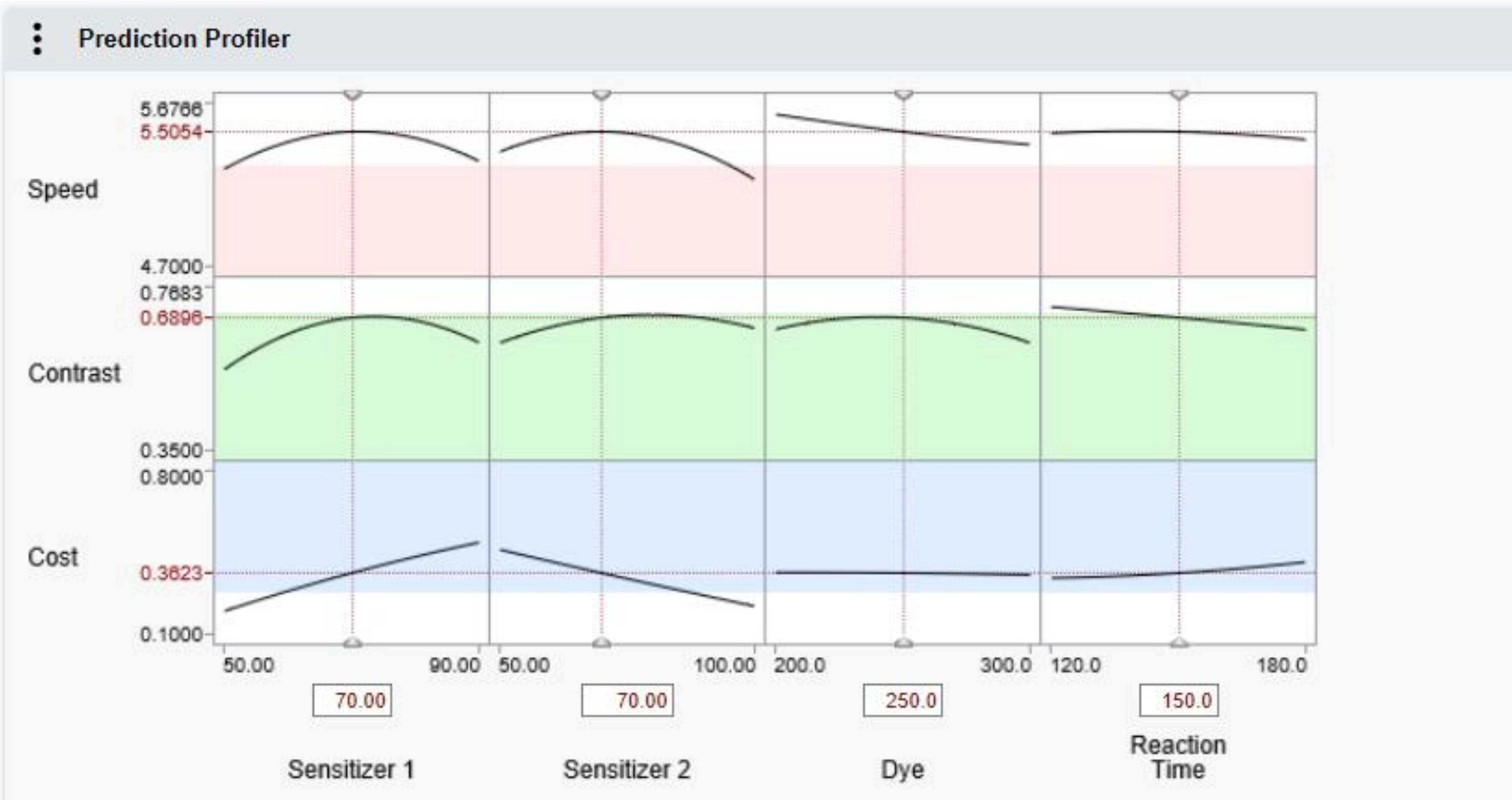


Trade-off & Optimization

Profiler after centering in acceptable region in contour plot



View/Share as Interactive HTML or on JMP Public or JMP Live



Remembered Settings

	Setting	Sensitizer 1	Sensitizer 2	Dye	Reaction Time	Speed	Contrast	Cost
Apply	Equal Importance Opt	80.753574	91.269729	250.57625	120	5.3542877	0.7466933	0.2504014
Apply	Mid Point Settings	70	70	250	150	5.5054353	0.6895831	0.3623274
Apply	Cost 6X Speed & Contrast	84.016038	93.725925	283.02514	120	5.2902084	0.72549	0.1991539
Apply	Opt Spd3X-Cntr1X-Cost6X	81.958309	90.706277	286.82246	120	5.3269582	0.7177857	0.2211116

View optimizations on your phone. Scan the QR code to launch browser, then use finger to interact with the Prediction Profiler and to "Apply" saved settings.

[Interactive HTML]

Broken Design Repair

- Create Pizza design
- Delete Hi-Hi and lo-Lo combinations of Time & Temp
- Add constraints and augment

Optimality comparison

A-optimality case study

Monte Carlo Power Calculations Add-in

https://testscience.org/skprjmp/

1 April 2024

Hi Tom,

We've completed the public release process for the Monte Carlo power JMP add-in: you can find the download link at the page below to play around with it.

https://testscience.org/skprjmp/

Best regards,
Tyler

Dr. Tyler Morgan-Wall
Institute for Defense Analyses
Operational Evaluation Division
Research Staff Member
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SIPR: Tyler.Morgan-Wall.ffrdc@ida.pentagon.smil.mil

STATISTICAL DISCOVERY

Mastering JMP

Recorded March 18, 2022

› BASIC

Optimizing Processes Using Designed Experiments

APPLICATION AREA:

Design of Experiments

Learn to use the six-step framework for creating a successful designed experiment to determine the relationship between the factors impacting a process and the optimal process output. See how to describe the goal, specify effects for an assumed model, generate a design, collect experimental run data, create and fit a model incorporating experimental data, and use factors to optimize and improve settings. See a 30-minute demo and stay on if you want to join 15-30 minutes of Topic Discussion and Q&A.

This session covers: definition of terms; using Custom Design tools to specify responses, factors and runs; understanding interactions; specifying models and model effects; evaluating models; and determining optimal factors for the process.

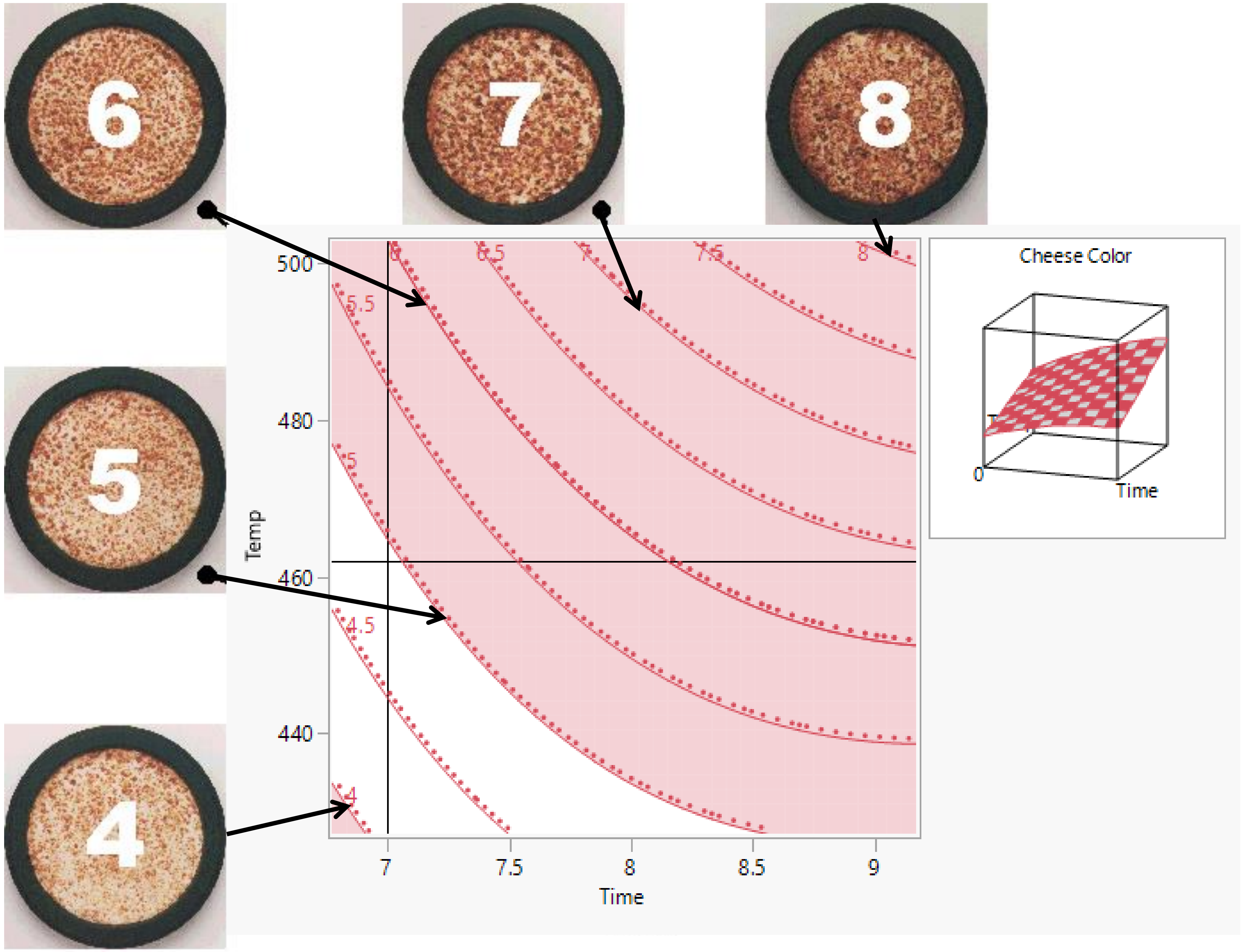
On-Demand Webinar

Watch Now

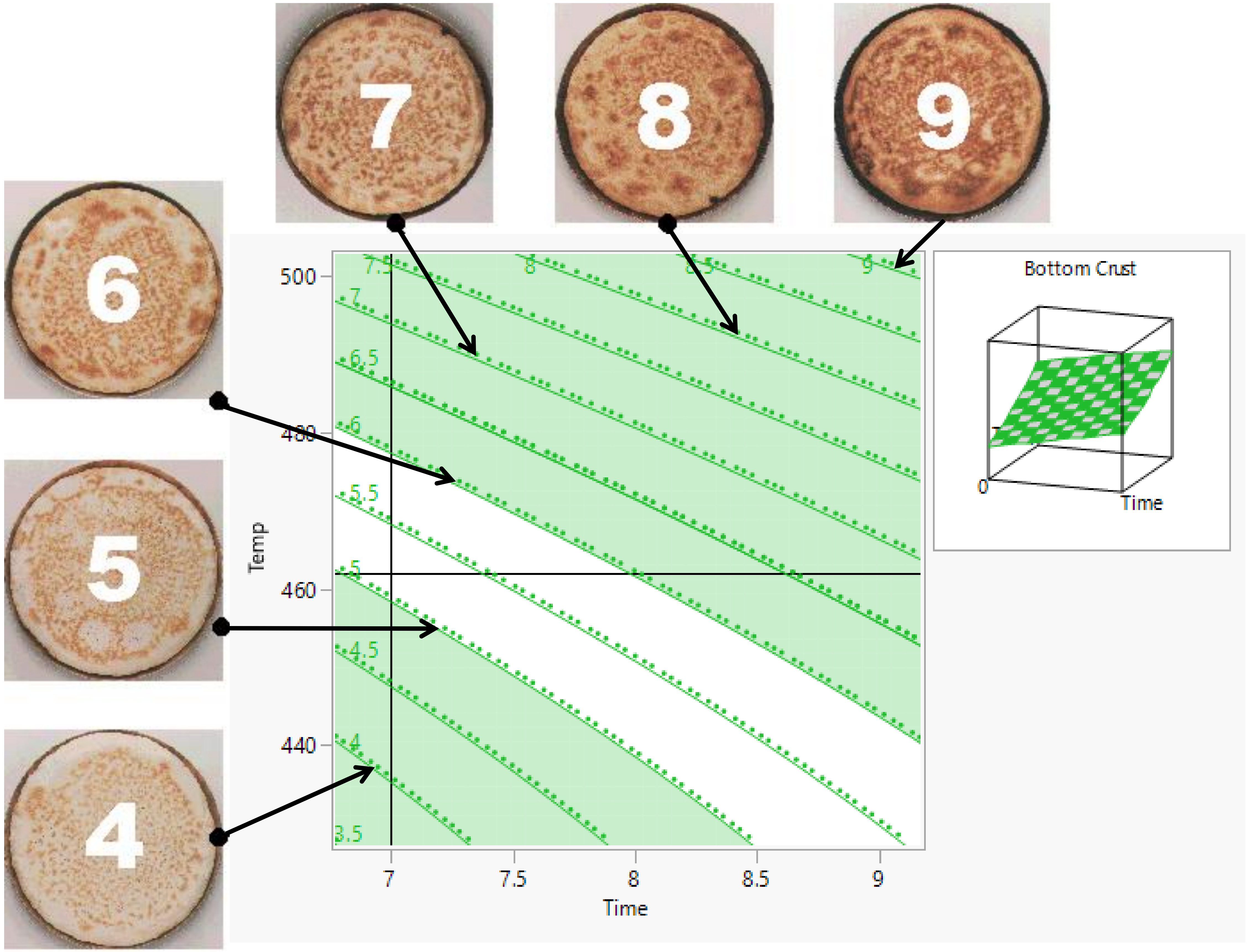
https://www.jmp.com/en_us/events/mastering/topics/optimizing-processes-using-doe.html

Cheese Color

Target Value 4.5 ± 0.5



Bottom Crust
Target Value 5.5 ± 0.5



Top Crust
Target Value 5.5 ± 0.5